A Dynamic Network Perspective on Resilient Control

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Plenary Lecture

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Colorado School of Mines

Located in Golden, Colorado, USA
10 miles West of Denver

CSM sits in the foothills of the Rocky Mountains

CSM has ~300 faculty and ~5600 students
(~4200 undergrad and ~1400 grad)

CSM is a public research institution devoted to engineering and applied science, especially:

- Discovery and recovery of **resources**
- Conversion of resources to **materials and energy**
- Utilization in advanced **processes and products**
- Economic and social systems necessary to ensure **prudent and provident use of resources in a sustainable global society**

Sustainable Global Society

CSM
PRO

- W
- Sc
- Al)
- Po tion res
- NS
- No ator)
- En
- Bi l
- Coursework in sustainable design
- Humanitarian Engineering minor; Engineering by Doing program

- **Comment:** Resilience, especially as related to infrastructure, is essential to a sustainable global society

- **Today:** Explore resilience from the perspective of controlling dynamic networks

Synopsis -1

- Many important systems can be modeled as
  - Collection or network of **integrating agents or subsystems**
  - **Exchanging energy, material, or information**
  - According to some **protocol or physical laws**
  - Subject to an **interconnection topology**

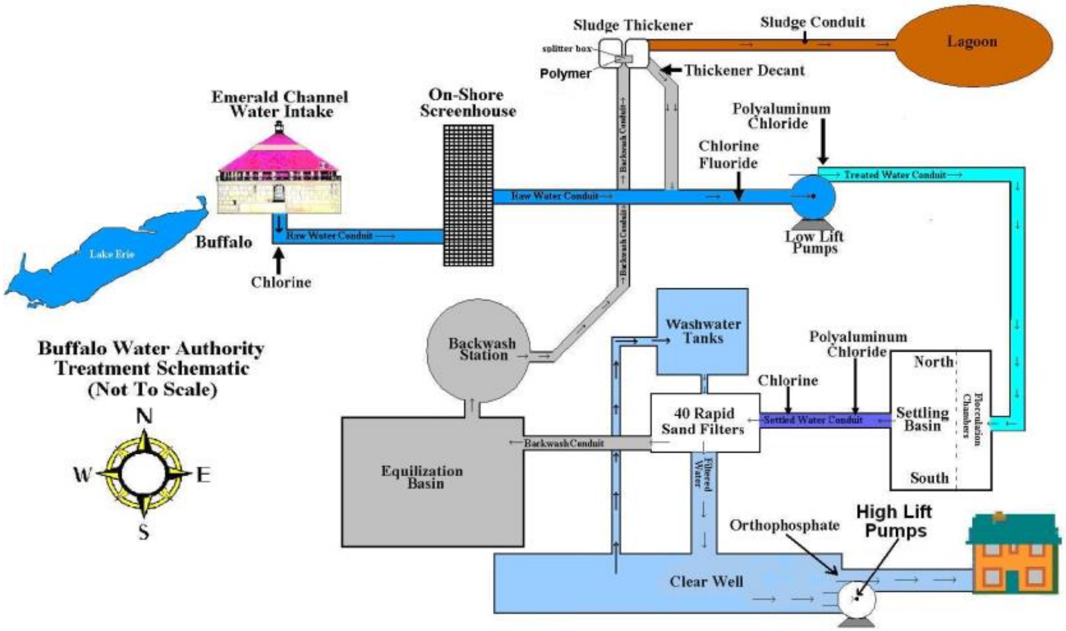


Figure: from http://www.buffalowater.org/files/Schematic-2011.JPG. Used without permission.

Example: Natural Gas System

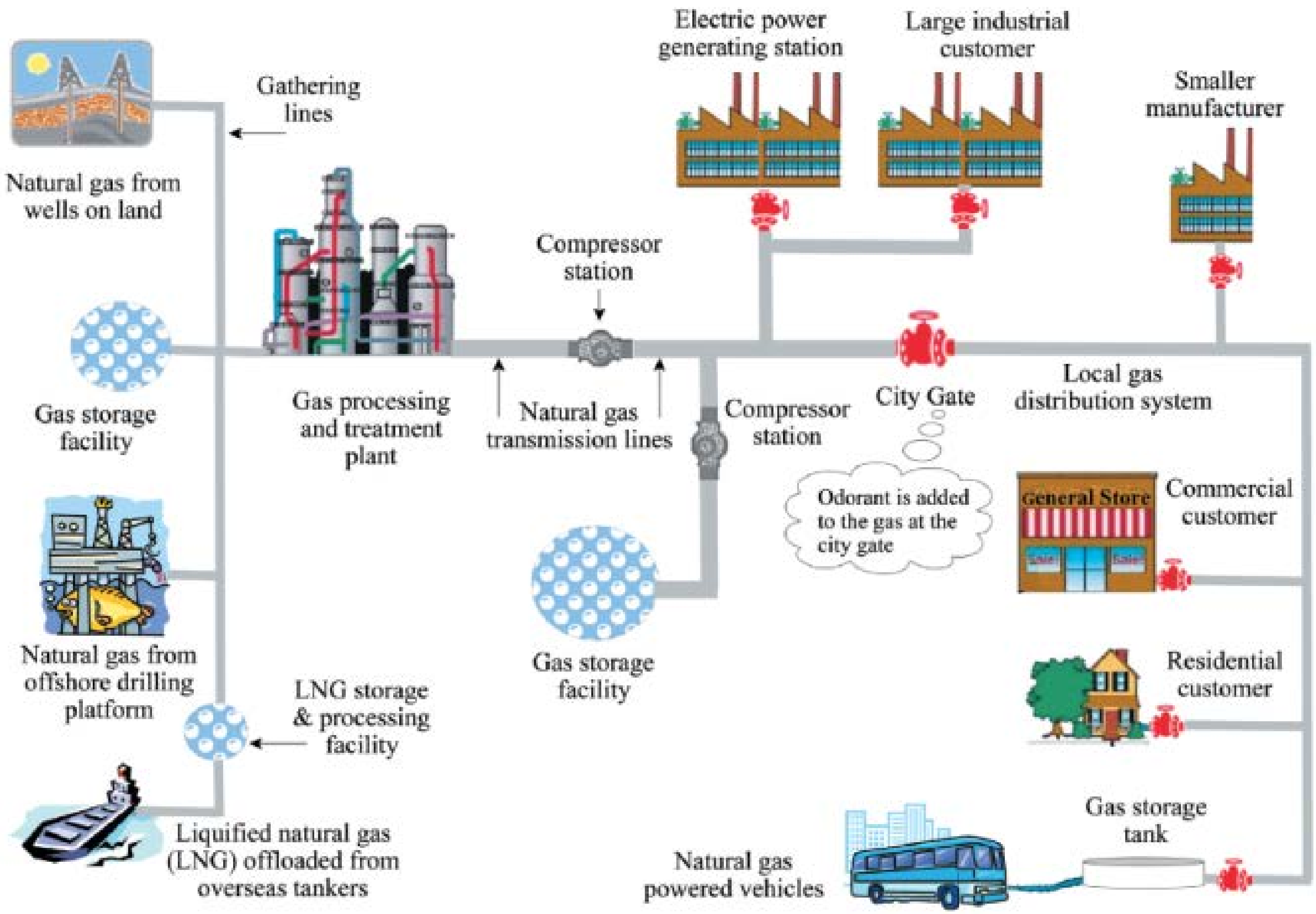
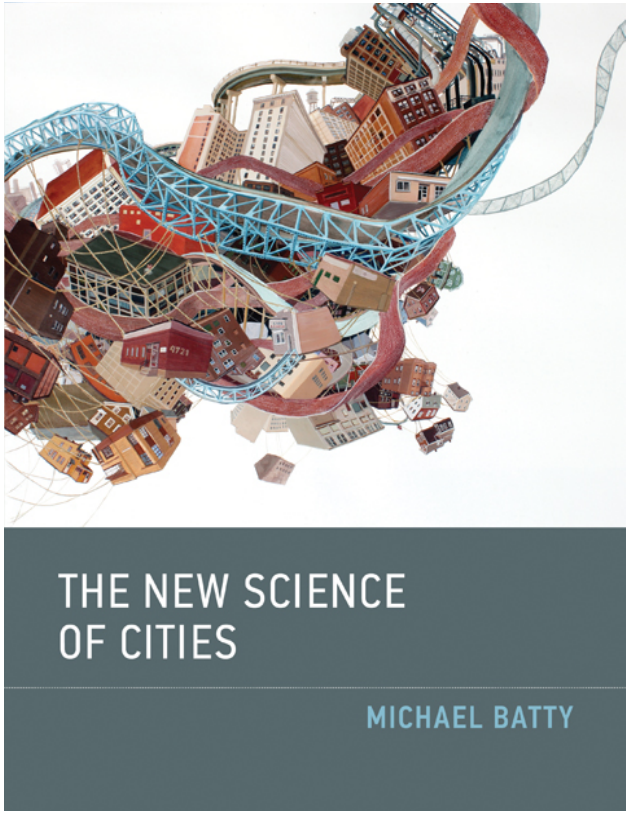


Figure: from Figure 256 in “From Reservoir to Burner Tip: A Primer,” Curtis and Schwochwow, in *Potential Supply of Natural Gas*, 2008. Used without permission.

Synopsis -2

- When such systems are governed by differential equations we call them a **dynamic network**
- Also called a **cyber-physical system** when there is
  - Tight integration of
    - Physical system dynamics
    - Sensors and actuators
    - Computing infrastructure
  - Multiple time and spatial scales
  - Multiple behavioral modalities
  - Context dependent interactions
- **Example**: Intelligent vehicle-highway system; cities



Example: Global Supply Chain

B Wible et al. Science 2014;344:1100-1103

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Port of Hong Kong

Science
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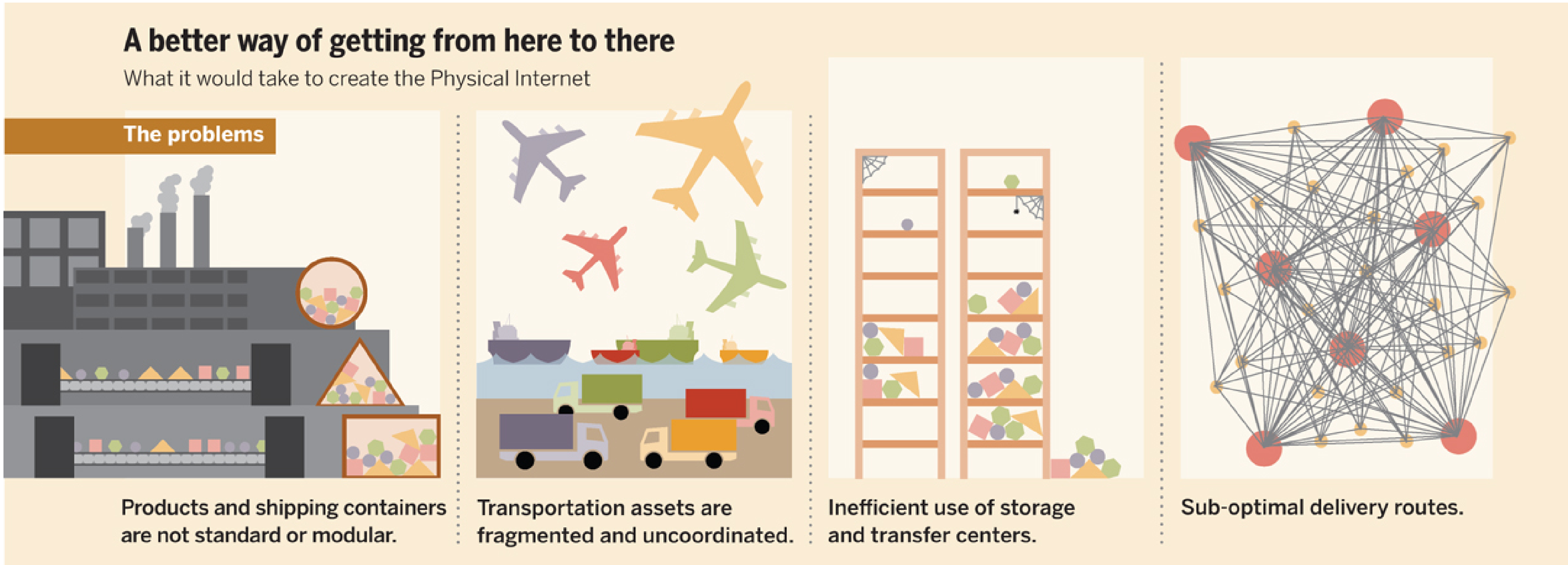
Logistics Today



Non-standard, non-modular shipping containers

Fragmented, un-coordinated transportation assets

Inefficient storage & transfer centers

Sub-optimal delivery routes

Figure from J Mervis Science 2014;344:1104-1107

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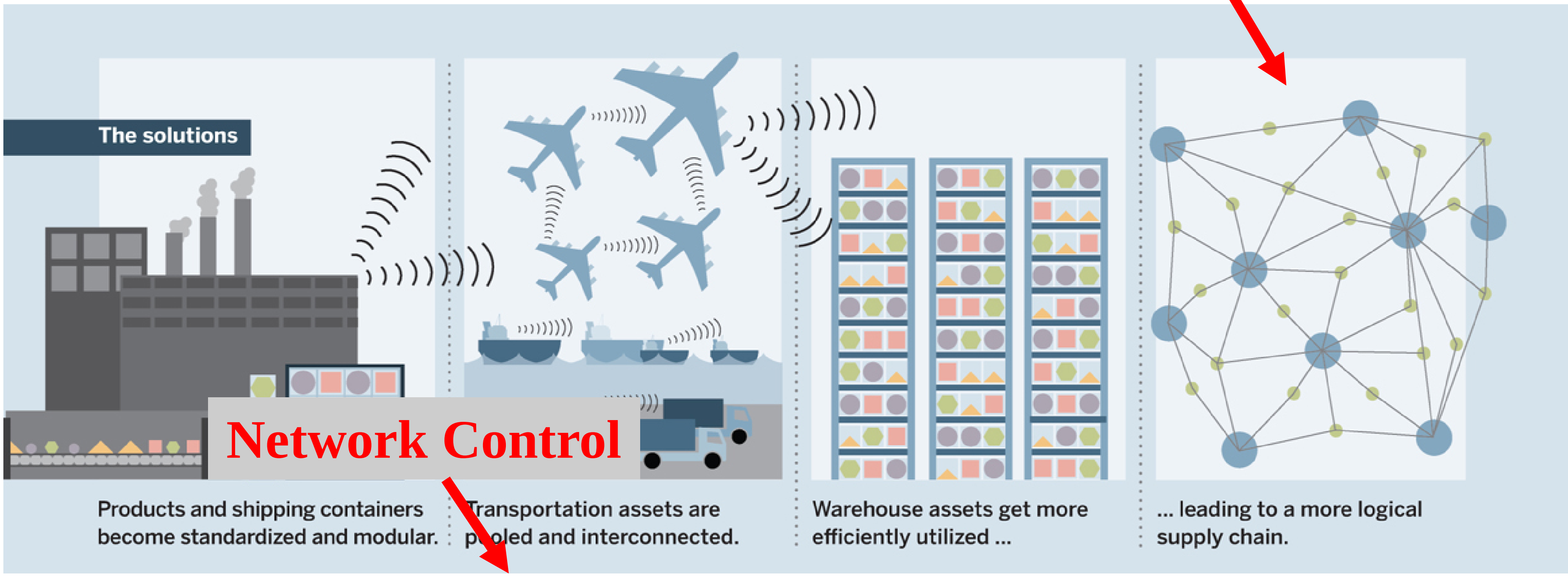
Logistics Tomorrow

Resilient Dynamic Network



Network Control

Standard, modular shipping containers

Pooled, interconnected transportation assets

Organized, coordinated warehouse assets

A more logical supply chain

Network Design

Figure from J Mervis Science 2014;344:1104-1107

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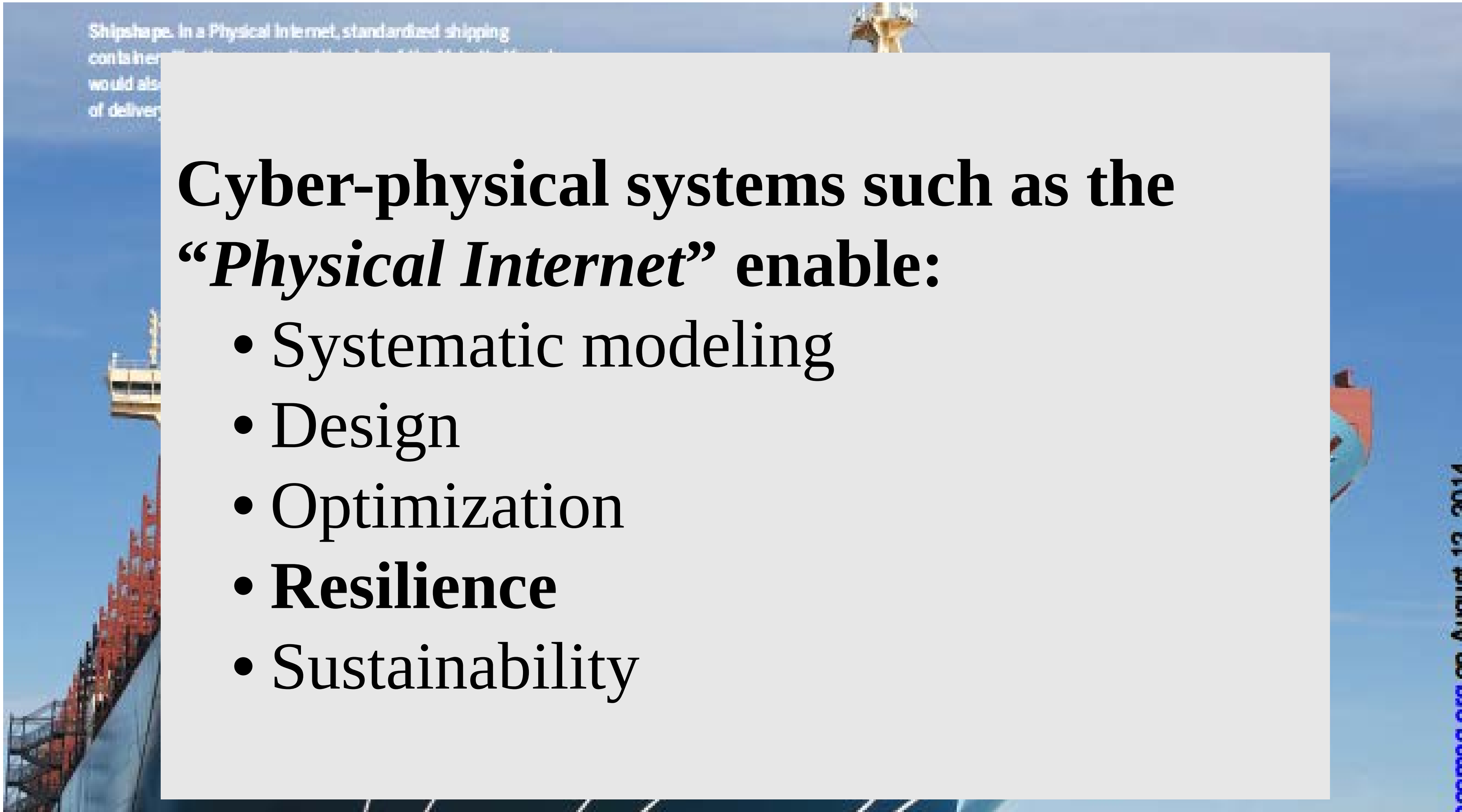
The Physical Internet



Cyber-physical systems such as the "*Physical Internet*" enable:

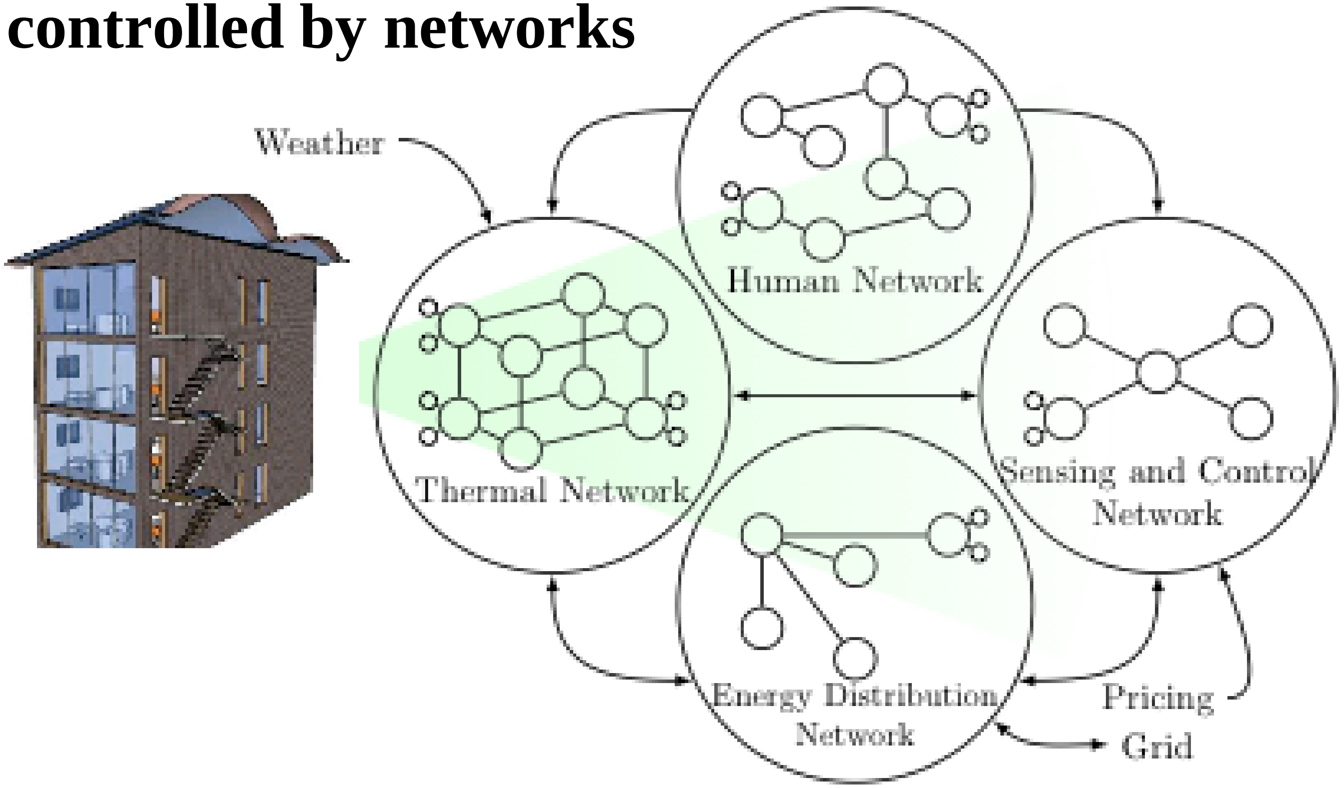
- Systematic modeling
- Design
- Optimization
- **Resilience**
- Sustainability

The information highway gets physical

The Physical Internet would move goods the way its namesake moves data

Synopsis -3

- Physical processes interacting with other sources of energy, material, and information suggests the interpretation:
  - **Complex infrastructure systems are networks controlled by networks**



Synopsis -4

- **In this talk**:
  - Apply “networks controlled by networks” idea to resilience
- **Approach**:
  - Model resilient control problem as disturbance or noise attenuation in dynamics consensus networks
- **Observation** will be:
  - Network topology matters
- **Comment** will be: future research needs to explore relationships between
  - Control-theoretic concepts
  - Graph-theoretic properties of networks

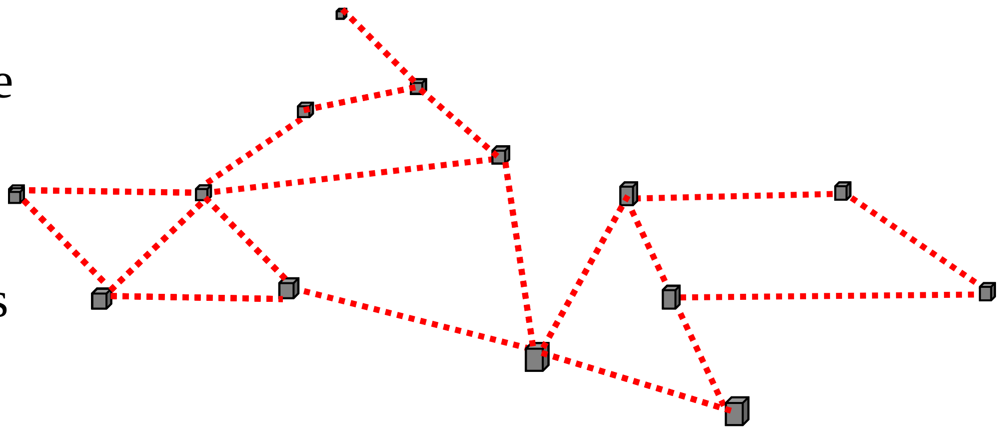
Outline

Introduction
- Systems as networks

Consensus Paradigm
- **Dynamic Networks**
- **Concepts and extensions**
- **Consensus and resilience**
- **Examples**

Resilient Dynamic Networks though Disturbance Attenuation
- Designing network weights
- Designing network controllers

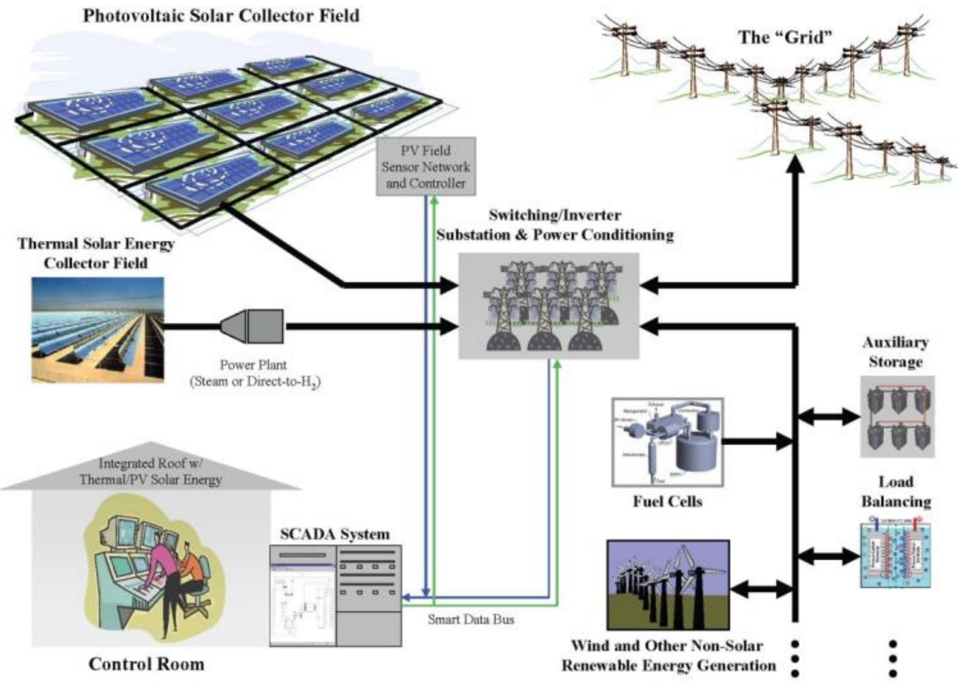
Dynamic Networks



- **Network of "entities"**
  - Communication infrastructure
  - Entity-level functionality
  - Implied global functionality
  - Not necessarily homogeneous
- **Nodes:**
  - Entities could be sensors
  - Entities could be actors (actuators)
  - Entities could be people
- **Dynamic**
  - Entities may or may not be mobile
  - Communication topology might be time-varying
  - Data actively and deliberately shared among entities
  - Decision-making and learning
  - Links between entities might be dynamic systems

Dynamic Networks as Models for …

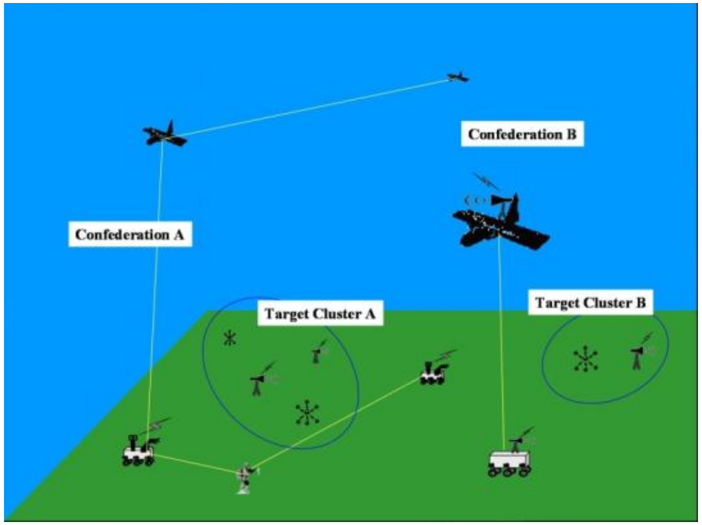
- **Many systems of interest:**
  - Cooperating robots
  - Buildings, cities
  - Power systems
  - Water distribution
  - Information networks
  - Socio-economic systems
  - …. many more ….

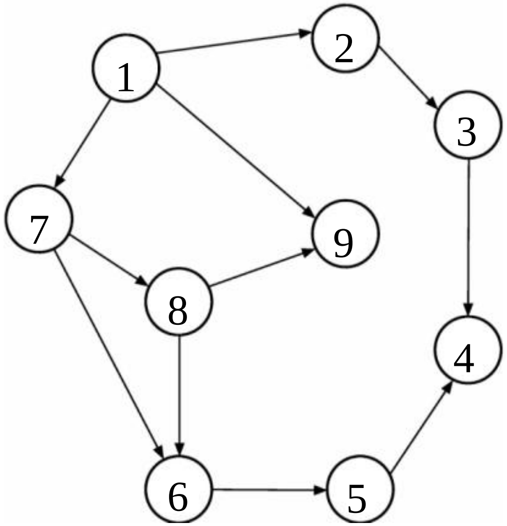


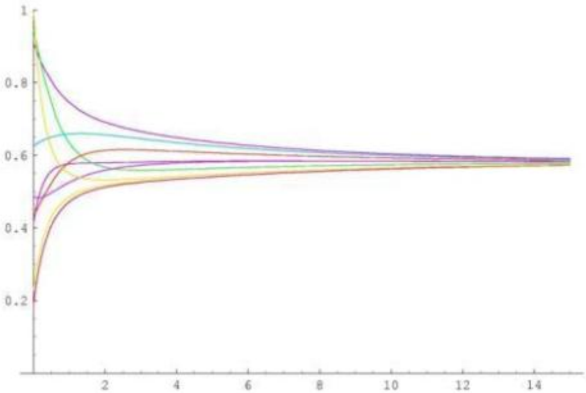
- Need a **framework for analysis and design** of these networks
  - One useful paradigm is the **consensus variable** approach

Consensus: an Algorithmic Approach to Coordination and Control in Networks

- The **consensus variable paradigm** is a generalization of potential field approaches and has connections to problems in:
  - Coupled-oscillator synchronization
  - Neural networks
- Also called agreement protocol
- Related to gossip algorithms
- Articulated in context of team theory in 1960s







Consensus Variable Perspective

- **Assertion:**
  - Multi-agent coordination requires that *some* information must be shared
- **The idea:**
  - Identify the essential information, call it the *coordination or consensus variable*.
  - Encode this variable in a distributed dynamical system and come to consensus about its value
- **Examples:**
  - Planning date and time and place of a meeting
  - Frequency control in power grid
  - Adaptive scheduling of mission timings

Consensus Variables

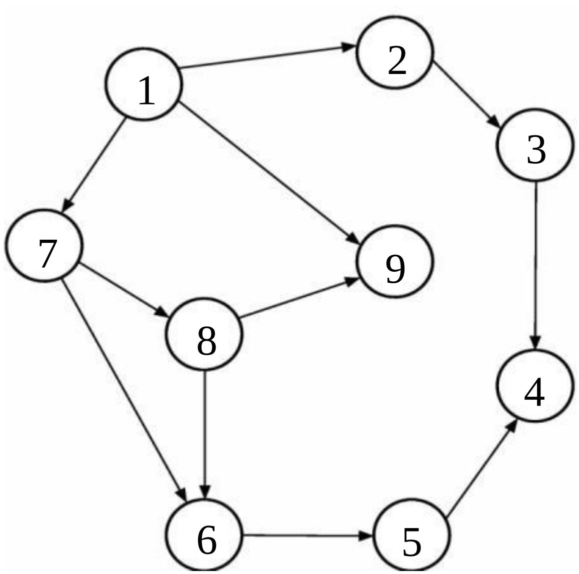
- Suppose we have *N* agents with a shared *global* consensus variable ξ
- Each agent has a *local* value of the variable given as ξ_i
- Ea **Change in value** eir local value based on the values of the agents that they can communicate wi **Difference with neighbors**

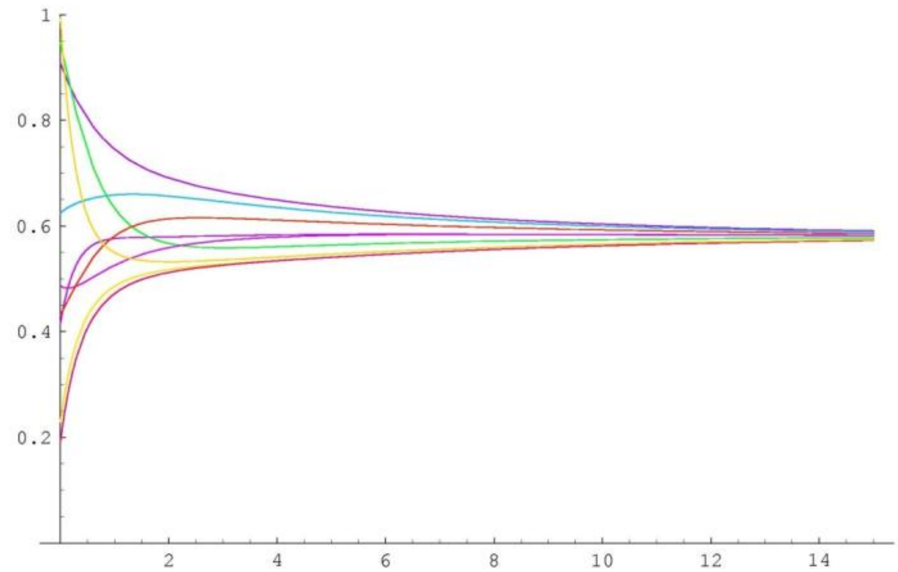
$$\dot{\xi}_i(t) = -\sum_{j=1}^{N} k_{ij}(t) G_{ij}(t) (\xi_i(t) - \xi_j(t))$$

where k_{ij} are gains and G_{ij} defines the communication topology graph of the system of agents

- **Key result** from literature: If the corresponding graph has a spanning tree then $\xi_i \longrightarrow \xi^*$ for all *i*

Example: Single Consensus Variable





$$
\begin{pmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \\ \dot{\xi}_4 \\ \dot{\xi}_5 \\ \dot{\xi}_6 \\ \dot{\xi}_7 \\ \dot{\xi}_8 \\ \dot{\xi}_9 \end{pmatrix}
=
\underbrace{\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
k_{21} & -k_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & k_{32} & -k_{32} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_{43} & -k_{43}-k_{45} & k_{54} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -k_{56} & k_{56} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -k_{67}-k_{68} & k_{67} & k_{68} & 0 \\
k_{71} & 0 & 0 & 0 & 0 & 0 & -k_{71} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & k_{87} & -k_{87} & 0 \\
k_{91} & 0 & 0 & 0 & 0 & 0 & 0 & k_{98} & -k_{91}-k_{98}
\end{bmatrix}}_{\text{Laplacian Matrix}}
\begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \\ \xi_7 \\ \xi_8 \\ \xi_9 \end{pmatrix}
$$

Extension 1 - Forced Consensus

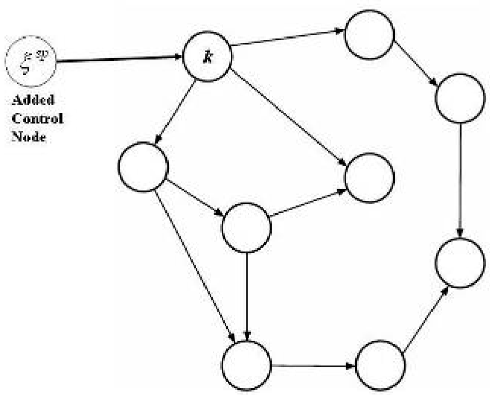
- Forced Consensus
  - Injecting an input into a node:

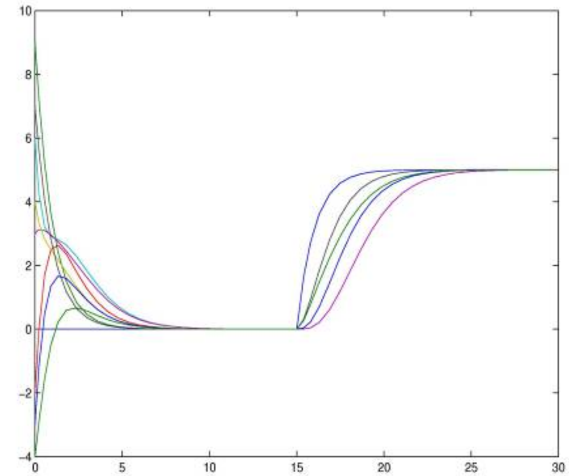
$$\dot{\xi}_i(t) = -\sum_{j=1}^{N} k_{ij}(t) G_{ij}(t)(\xi_i(t) - \xi_j(t)) + b_i u_i$$

  - Then we use a feedback controller:

$$u_i(t) = k_p(\xi^{sp} - \xi_i)$$

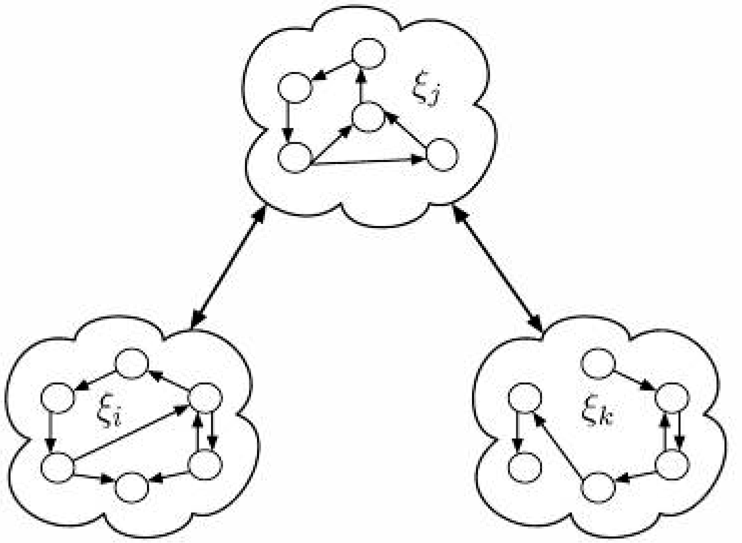
- Example:





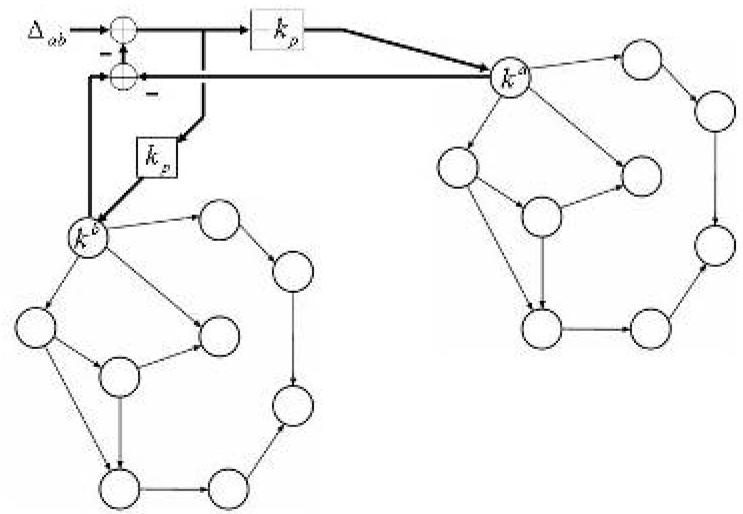
Extension 2 – Multiple, Constrained Consensus

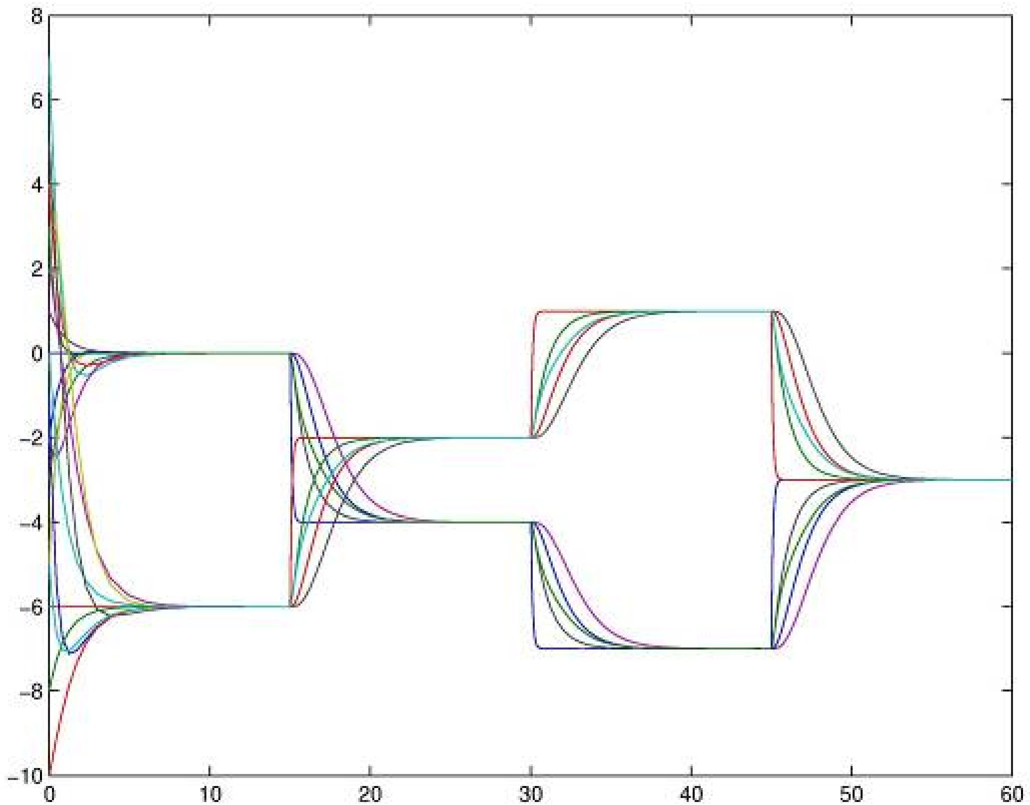
- Often we will have multiple consensus variables in a given problem



- It can be useful to enforce constraints between these variables, specifically, to have $\xi_i = \xi_j + \Delta_{ij}$
- Again we can give a feedback control strategy to achieve this type of constrained consensus between groups of agents

Example – Multiple, Constrained Consensus





Extension 3 – Higher-Order Consensus

- **Example: Flocking and Formation Flight**
- Consider a third-order consensus problem, applied to a formation control problem with five vehicles
- One vehicle has acceleration setpoint input and is the leader

$$\dot{x}_i = v_i$$

$$\dot{v}_i = a_i$$

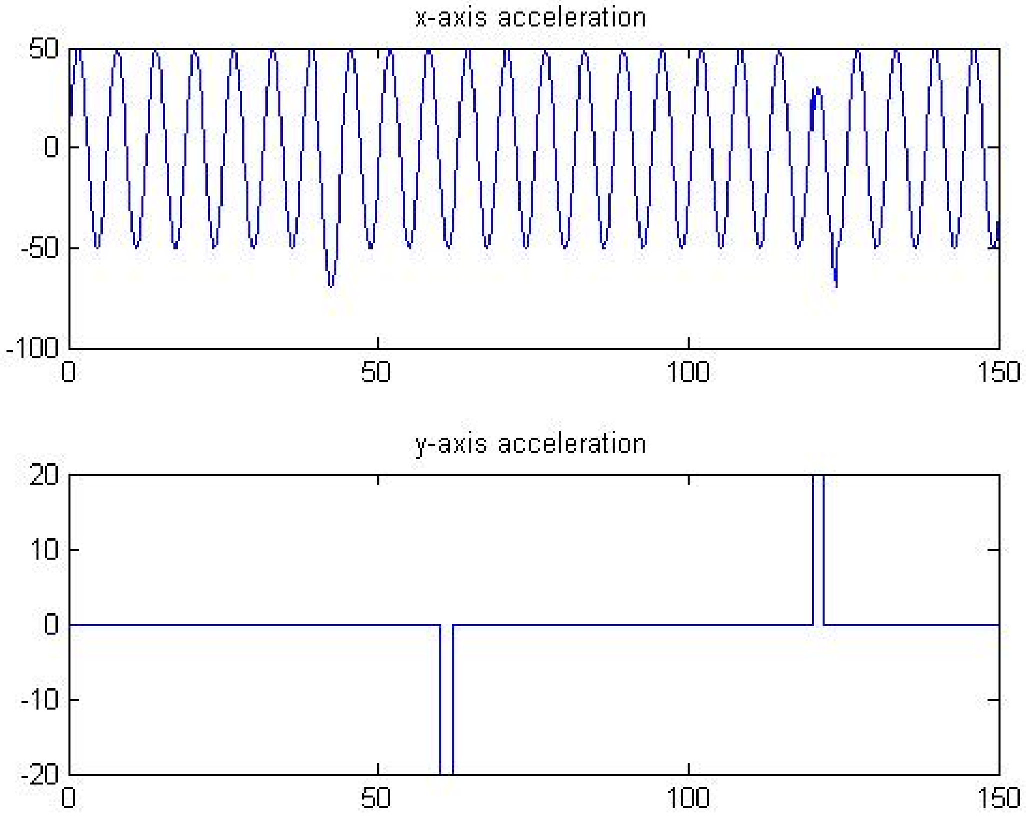
$$\dot{a}_i = -\sum_{j=1}^{n} g_{ij} k_{ij} \{ \gamma_0 [(x_i - \delta_i) - (x_j - \delta_j)] + \gamma_1 (v_i - v_j) + \gamma_2 (a_i - a_j) \} - \alpha (a_i - a_i^*)$$

Enables formation control

Acceleration Input

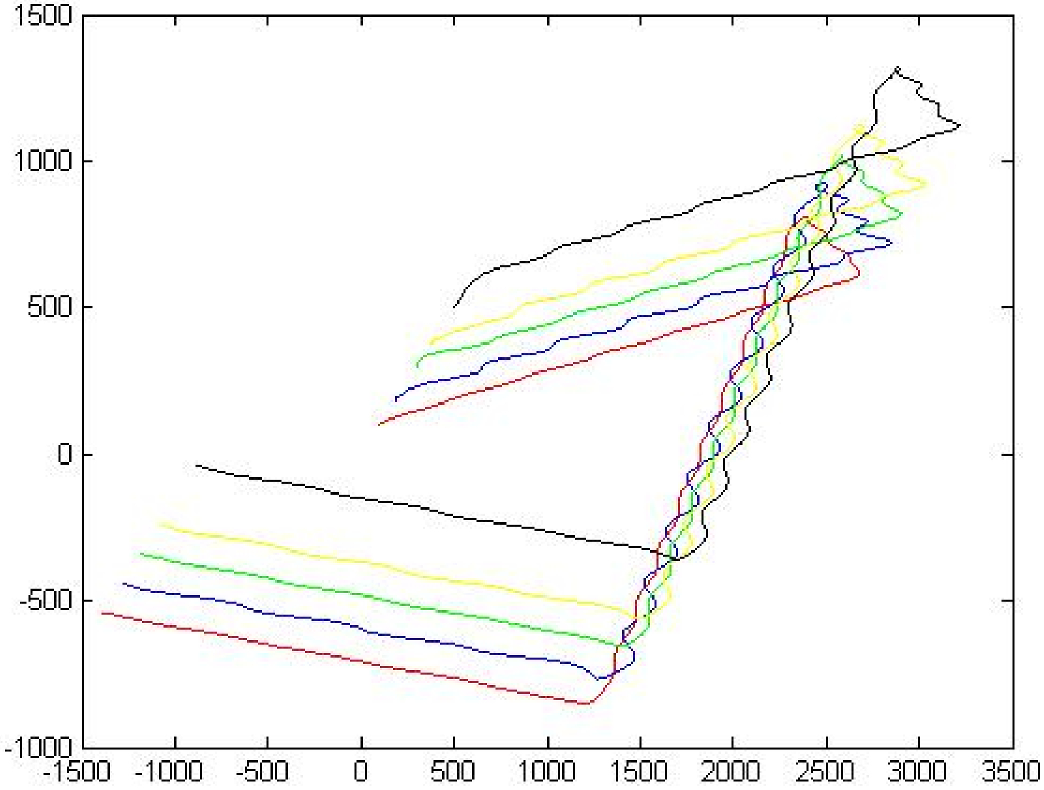
Extension 3 – Higher-Order Consensus

- The "leader node" sees the following acceleration input profile:



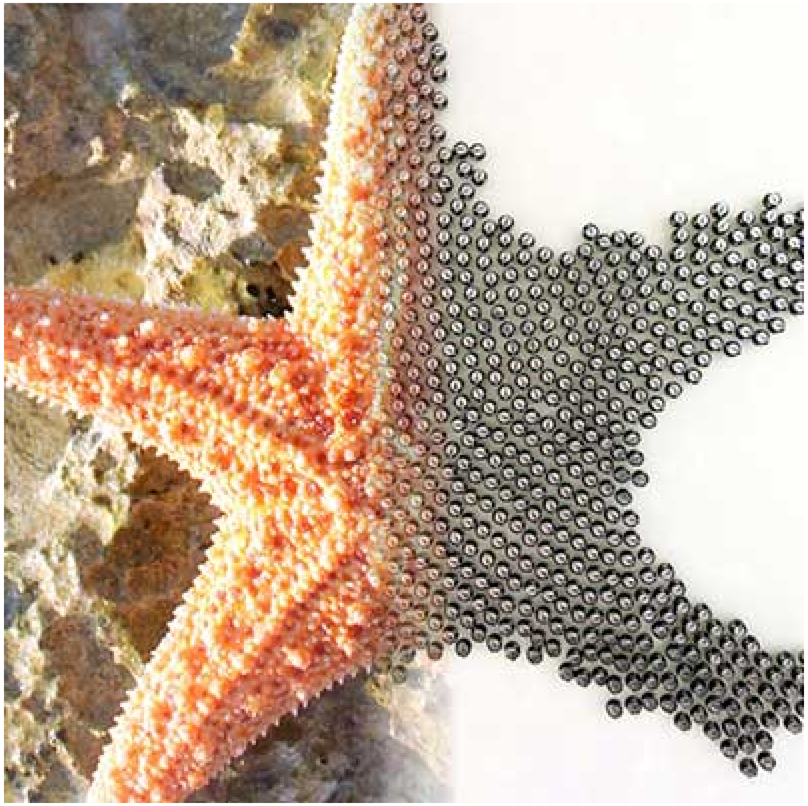
Extension 3 – Higher-Order Consensus

- The resulting paths look like:

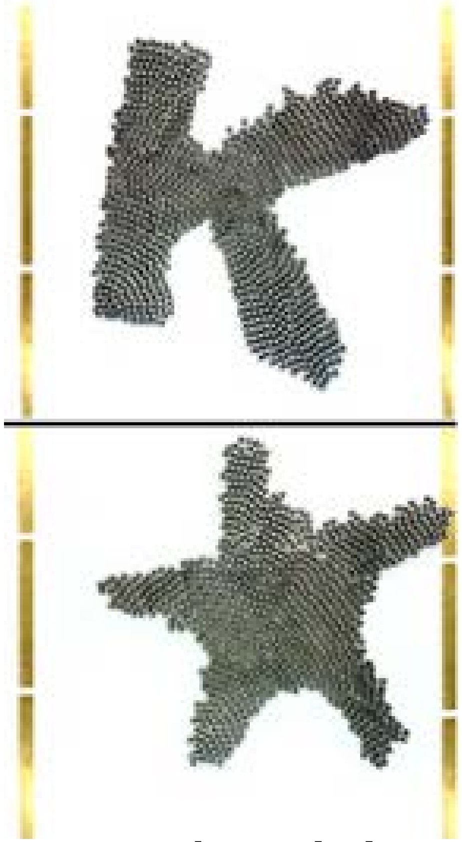


Consensus for Real -1: Harvard 1000-Robot Swarm

- *Science* 15 August 2014: Vol. 345 no. 6198 pp. 795-799



http://www.seas.harvard.edu/sites
/default/files/images/news/Image1_sq_0_op.jpg

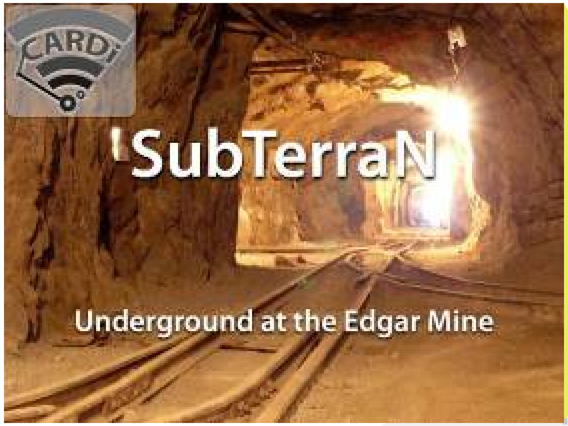


http://www.seas.harvard.edu/sites/default/fil
es/images/news/Image2_650.jpg

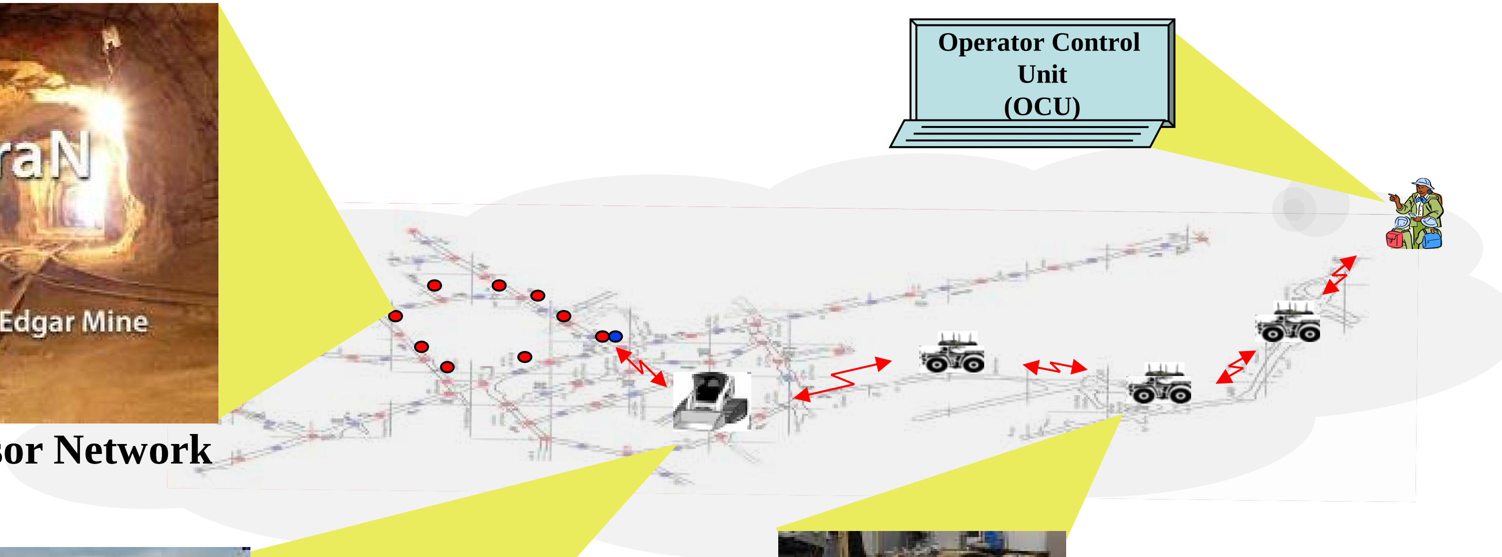
Consensus for Real -2: Radio Tethering in Subterranean Environments

- Subways, mines, caves, underground buildings
- Limited
  - Entrances/exits
  - Navigation
  - Limited ventilation
  - Communications
- Challenging emergency management environment
  - Assume no infrastructure
  - Radio relays may be necessary at/near junctions
  - Rescue workers must carry their own comms

MineSENTRY - Autonomous Mobile Radio Relays



Underground Sensor Network



Teleoperated Bobcat

Mesh Radio System
(Rajant Breadcrumb™)

Autonomous Radio Node (AMR)

- Provides Communication Tether
- Uses CSM-developed UGV Autopilot

Theoretical Approach

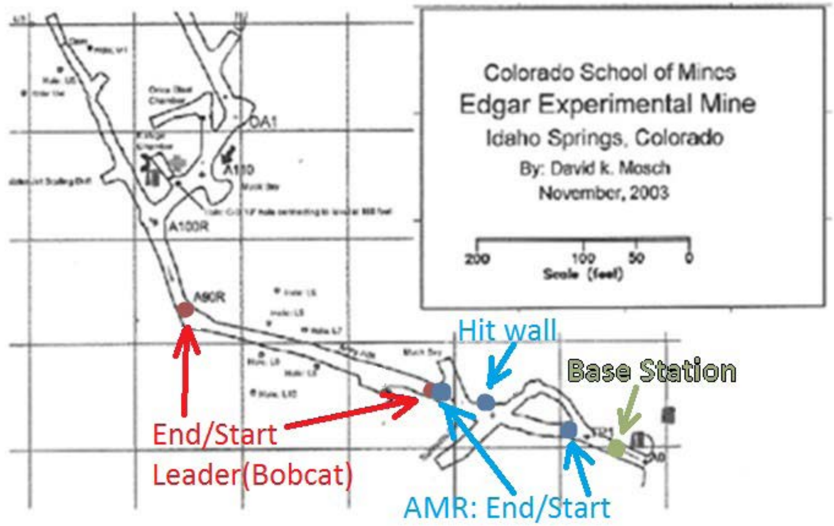
- Wireless 1-D tethering
  - Not in physical coordinates
  - Rather, in radio signal strength (RSS) space
- Goal is to maintain equal RSS between ARMs while the leader moves forward in the mine
- Our approach uses the **internal model principle to develop a higher-order (2nd) consensus algorithm**:

$$\ddot{x}_i = -k_i^p RSS^{dB}{}_{i+1,i} + k_i^p RSS^{dB}{}_{i,i-1} - k_i^d \frac{d}{dt}(RSS^{dB}{}_{i+1,i}) + k_i^d \frac{d}{dt}(RSS^{dB}{}_{i,i-1})$$

Experimental Results

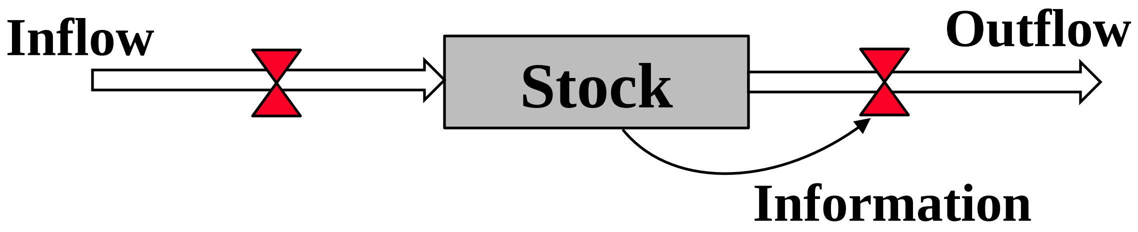
MineSENTRY

Mine Safety and Rescue through Sensing Networks and Robotics Technology



Consensus Networks and Resilience -1

- Dynamic consensus networks give a reasonable paradigm for modeling systems that have
  - Storage and computation at nodes
  - Flows between edges and along edges
- One possible way to see this is to consider the ideas from Jay Forrester's (MIT) System Dynamics paradigm
  - Basically a "poor man's control theory"
  - Envisioned all systems as having "stocks" and "flows" that are interconnected through positive and negative feedback



Consensus Networks and Resilience -2

- Stocks and flows and some other components can be assembled to build up complex systems models
- These models can be simulated using tools such as STELLA

Example: Public Policy Process

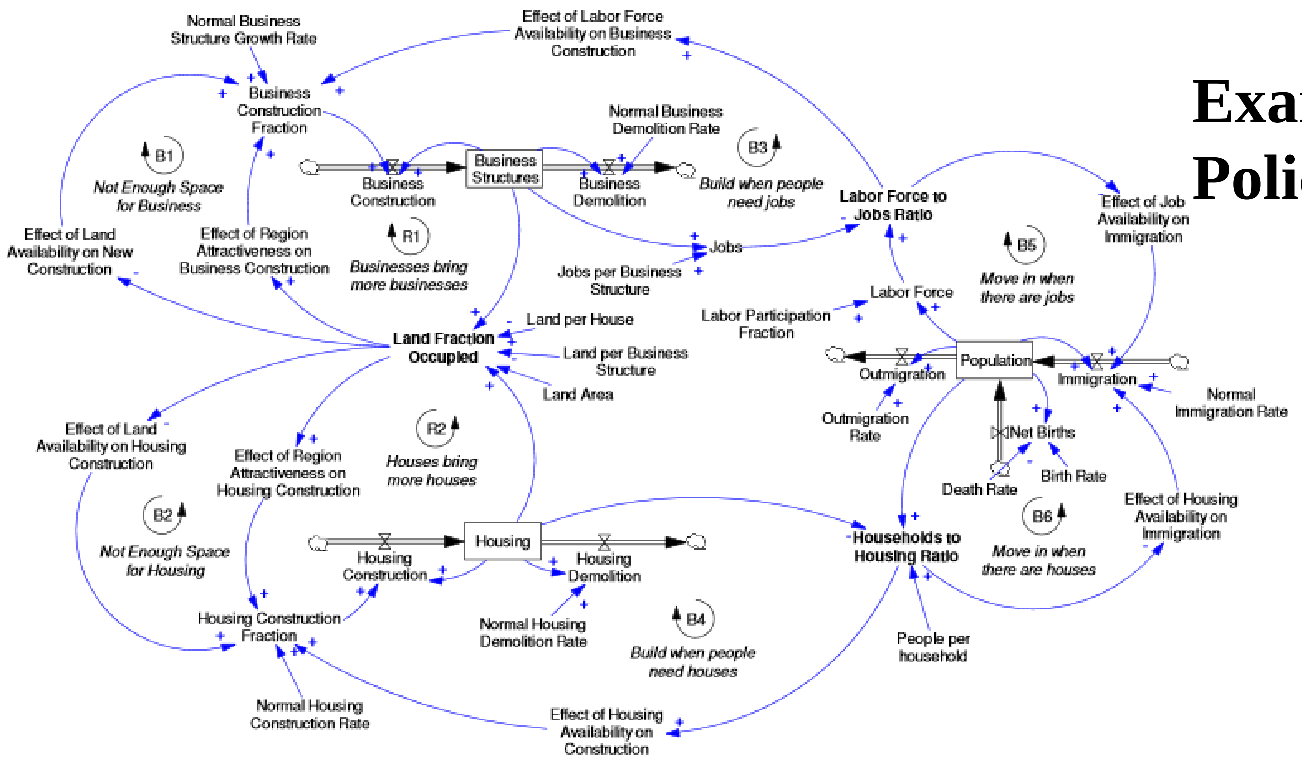
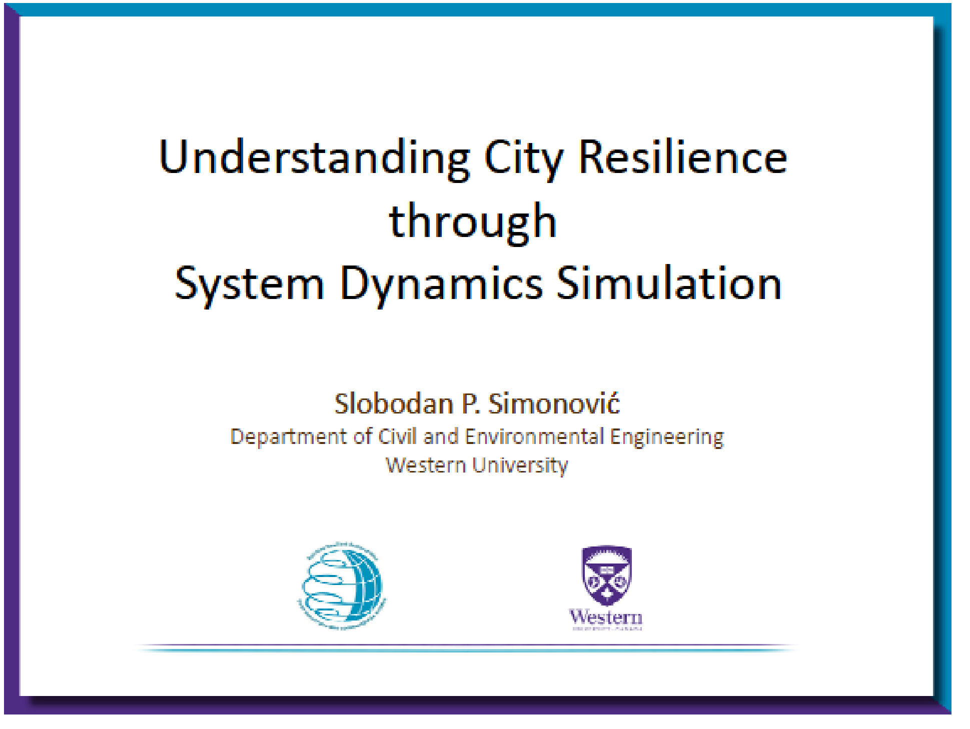


Figure: From "How Small System Dynamics Models Can Help the Public Policy Process," http://www.albany.edu/~gpr/SmallModels.pdf. Used without permission.

Consensus Networks and Resilience -3

- People have used these ideas to study resilience and sustainability. For example:



2012 Advanced Institute, Taipei
Slobodan P. Simonović

Consensus Networks and Resilience -4

- Consensus paradigm provides an analytical tool for analysis of systems modeled using Forrester's System Dynamics

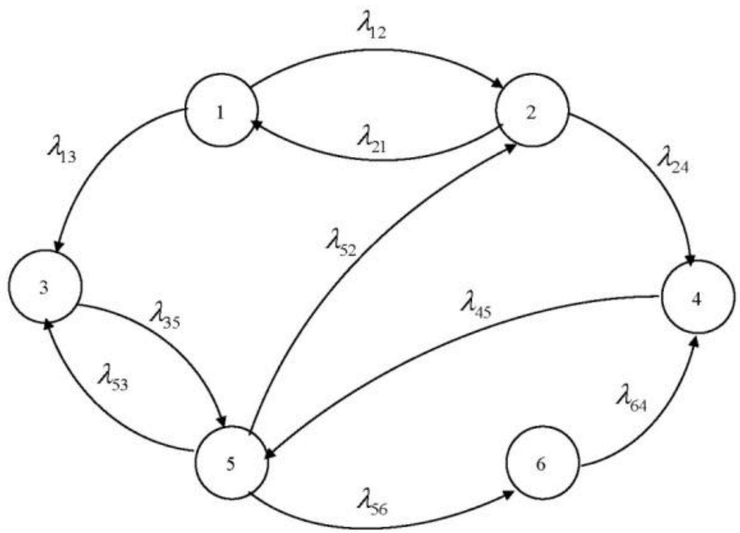
- Key idea is that it works for systems where

$$\textit{change in storage} \propto \sum \; (\textit{multipliers}) \; x \; (\textit{flows in} - \textit{flows out})$$

- Below we will illustrate this for several systems:
  - Cooperating robots (discussed above)
  - Thermal systems (e.g., buildings)
  - Electric circuits (e.g., power systems)

Static Graphs and Laplacian Matrix

- The neighbors of node n_i are $\mathcal{N}_i = \{j : (n_i, n_j) \in \mathcal{E}\}$
- The Laplacian matrix $L = [l_{ij}]$ is defined by:



$$l_{ij} = \begin{cases} \sum_{k \in \mathcal{N}_i} \lambda_{ik} & i = j \\ -\lambda_{ij} & i \neq j \text{ and } (i,j) \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases}$$

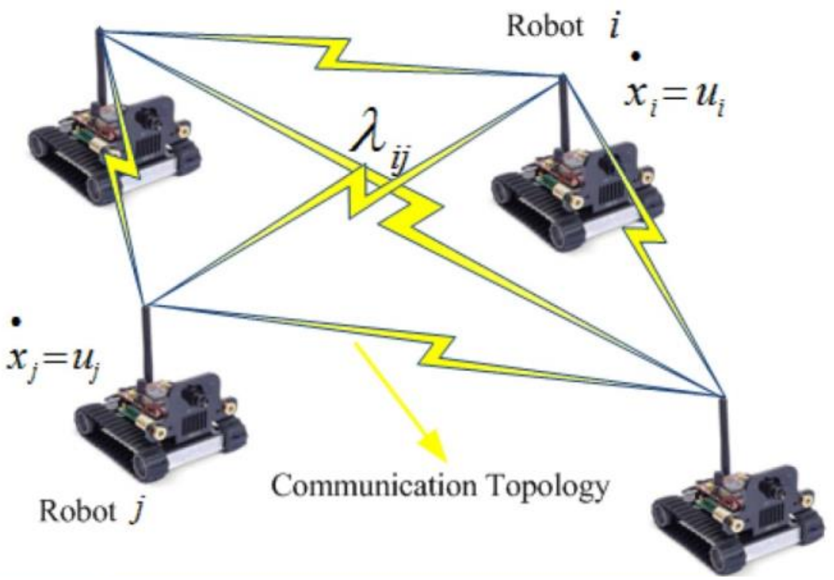
$$L = \begin{bmatrix} \lambda_{12} + \lambda_{13} & -\lambda_{12} & -\lambda_{13} & 0 & 0 & 0 \\ -\lambda_{21} & \lambda_{21} + \lambda_{24} & 0 & -\lambda_{24} & 0 & 0 \\ 0 & 0 & \lambda_{35} & 0 & -\lambda_{35} & 0 \\ 0 & 0 & 0 & \lambda_{45} & -\lambda_{45} & 0 \\ 0 & -\lambda_{52} & -\lambda_{53} & 0 & \lambda_{52} + \lambda_{53} + \lambda_{56} & -\lambda_{56} \\ 0 & 0 & 0 & -\lambda_{64} & 0 & \lambda_{64} \end{bmatrix}$$

Static Consensus Protocols

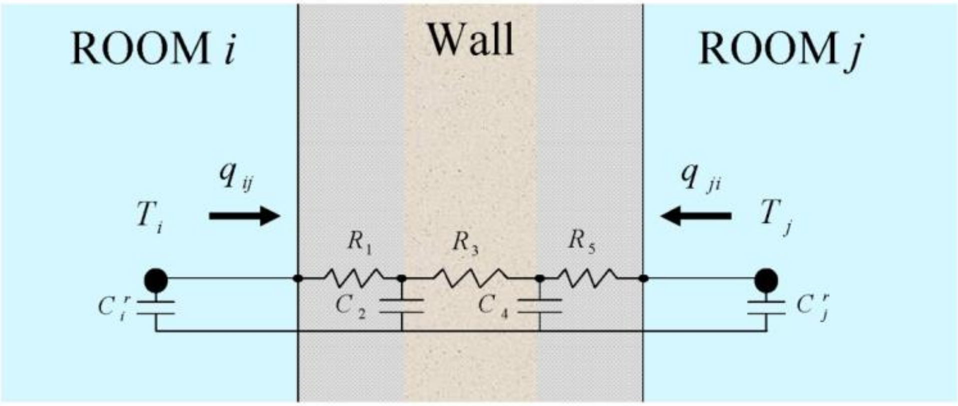
- Consider 4 robots with the velocity $\dot{x}_i = u_i$ and interconnected using the following static consensus protocol:

$$\dot{x}_i = u_i = -\sum_{j \in \mathcal{N}_i} [\lambda_{ij}(x_i - x_j)].$$

Then $$\dot{X} = -LX,$$
where L is the static Laplacian associated with the communication topology of the 4 robots graph.



Two Rooms Modeled as Two Interconnected Nodes

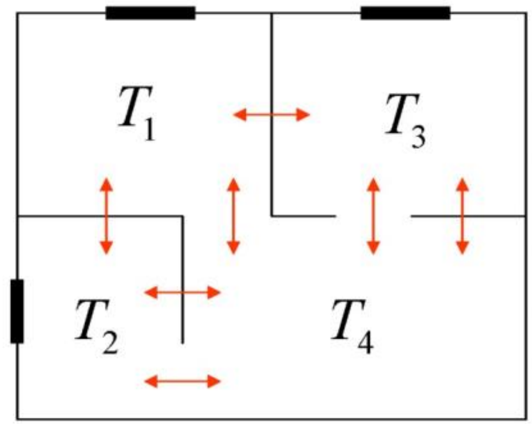


- The node equation can be written as: $C_i^r \frac{dT_i}{dt} = q_i^{in} - q_{ij}(t)$
- The heat flows can be written as:

$$\begin{bmatrix} q_{ij} \\ q_{ji} \end{bmatrix} = \frac{1}{B_{ij}(s)} \begin{bmatrix} A_{ij}(s) & -D_{ij}(s) \\ -D_{ij}(s) & A_{ji}(s) \end{bmatrix} \begin{bmatrix} T_i \\ T_j \end{bmatrix}$$

- $A_{ij}, A_{ji}, B_{ij}, D_{ij}$ are transfer functions (poylnomials in s), representing physical dynamics (ultimately described by a differential equation)

Hypothetical Four-Room Example



- For each room use

$$C_i^r \frac{dT_i}{dt} = q_i^{in}(t) - \sum_{j \in \mathcal{N}_i} q_{ij}(t)$$

or

$$sT_i(s) = \frac{1}{C_i^r} Q_i^{in}(s) - \sum_{j \in \mathcal{N}_i} [\lambda_{ij}^S(s) T_i(s) - \lambda_{ij}^C(s) T_j(s)]$$

- Define the vectors

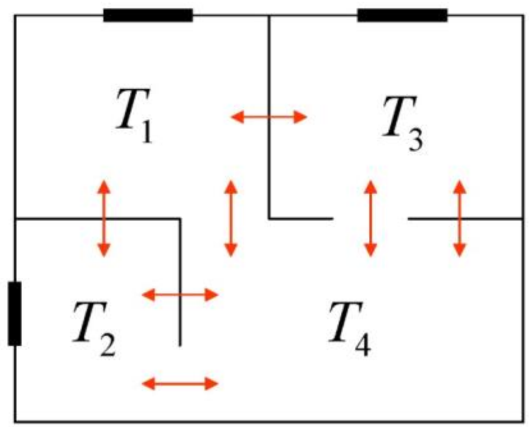
$$T(s) = \begin{bmatrix} T_1(s) & T_2(s) & T_3(s) & T_4(s) \end{bmatrix}^T$$

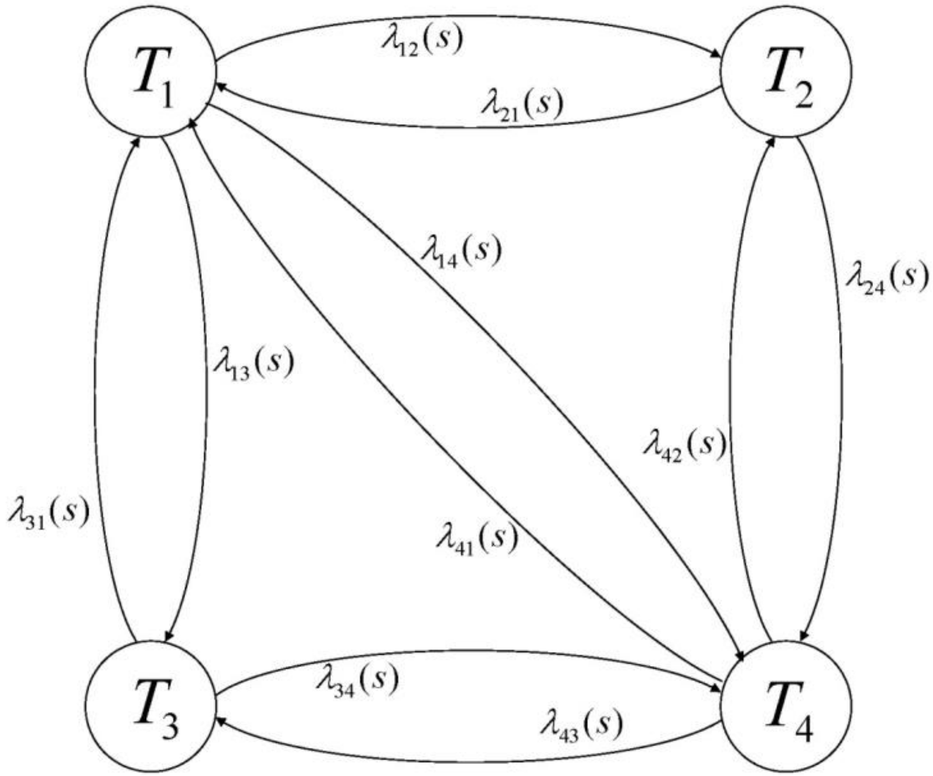
$$Q^{in}(s) = \begin{bmatrix} Q_1^{in}(s) & Q_2^{in}(s) & Q_3^{in}(s) & Q_4^{in}(s) \end{bmatrix}^T$$

- Then we can write $T(s) = \frac{1}{s}[Q^{in}(s) - L(s)T(s)]$, where

$$L(s) \quad = \quad \underbrace{\begin{bmatrix} \sum\limits_{j=2,3} \lambda_{1j}^S(s) & -\lambda_{12}^C(s) & -\lambda_{13}^C(s) & 0 & 0 & 0 \\ -\lambda_{21}^C(s) & \sum\limits_{j=1,4} \lambda_{2j}^S(s) & 0 & -\lambda_{24}^C(s) & 0 & 0 \\ 0 & 0 & \lambda_{35}^S(s) & 0 & -\lambda_{35}^C(s) & 0 \\ 0 & 0 & 0 & \lambda_{45}^S(s) & -\lambda_{45}^C(s) & 0 \\ 0 & -\lambda_{52}^C(s) & -\lambda_{53}^C(s) & 0 & \sum\limits_{j=2,3,6} \lambda_{5j}^S(s) & -\lambda_{56}^C(s) \\ 0 & 0 & 0 & -\lambda_{64}^C(s) & 0 & \lambda_{64}^S(s) \end{bmatrix}}_{\text{Dynamic Laplacian Matrix}}$$

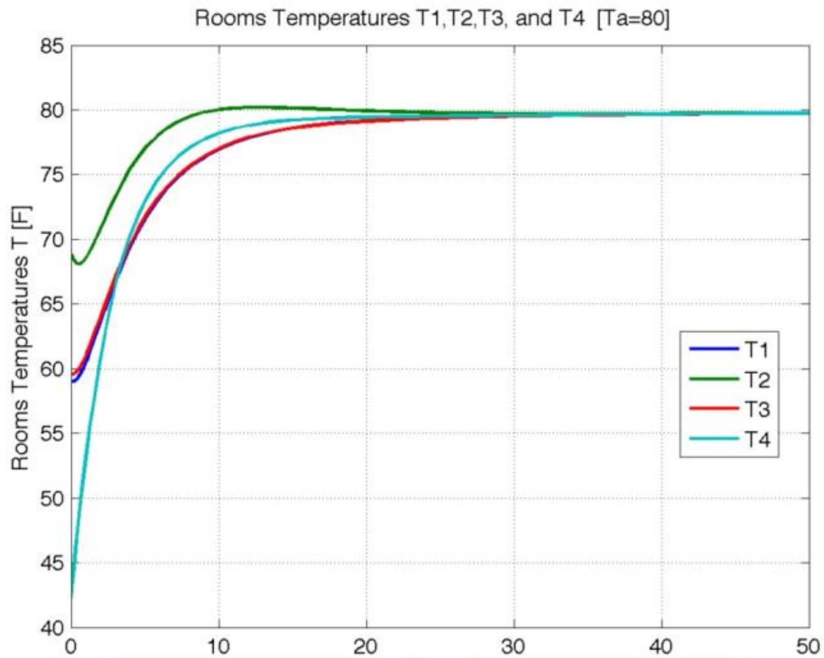
Hypothetical Four-Room Example as a Graph





Simulation: Hypothetical Four-Room Example

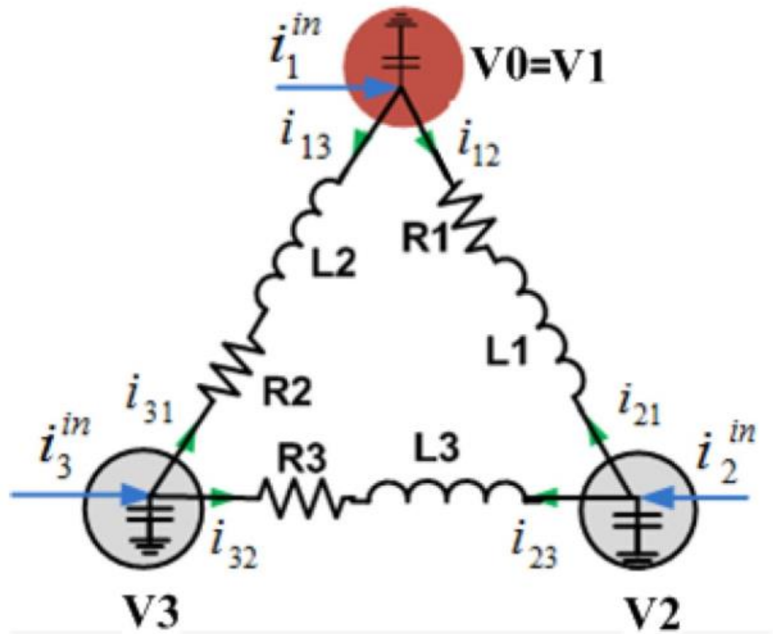
- Consider case with: (1) an outside node representing ambient conditions (dynamic consensus with a leader), with $T_a = 80$; (2) no other input energy; and (3)rooms initially set to arbitrary temperatures (less than ambient)
- As expected, all temperatures converge to the ambient temperature



Electrical Network as an Undirected Dynamic Consensus Network

- The dynamic model of each node is:

$$sV_i(s) = I_i^{in}(s) - \sum_{j \in \mathcal{N}_i} [Y_{ij}(s)(V_i(s) - V_j(s))]$$



- The dynamic consensus protocol:

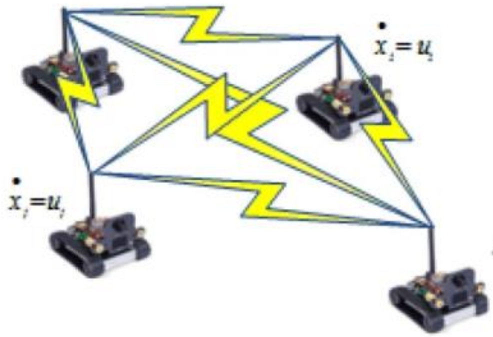
$$V_i(s) = -\frac{1}{s} \sum_{j \in \mathcal{N}_i} [Y_{ij}(s)(V_i(s) - V_j(s))]$$

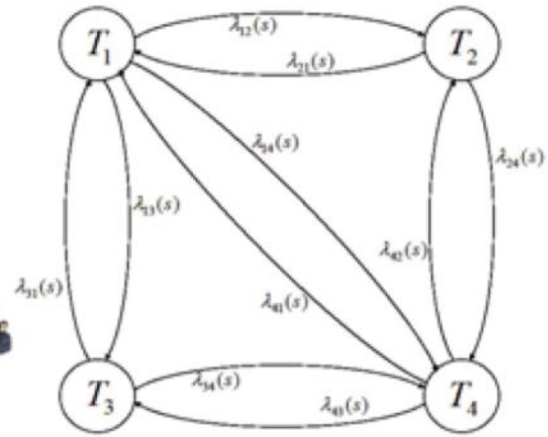
- The overall system:

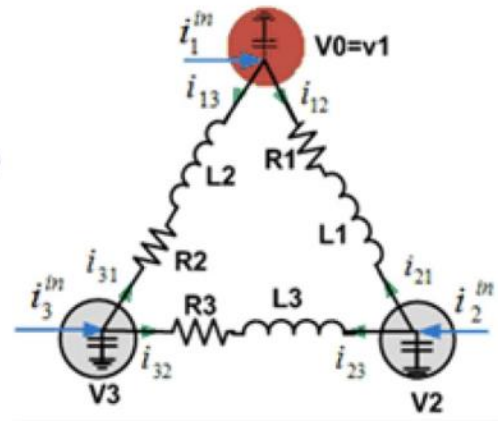
$$sV(s) = -L(s)V(s)$$

$$L(s) = \begin{bmatrix} Y_{12}(s) + Y_{13}(s) & -Y_{12}(s) & -Y_{13}(s) \\ -Y_{12}(s) & Y_{12}(s) + Y_{23}(s) & -Y_{23}(s) \\ -Y_{13}(s) & -Y_{23}(s) & Y_{13}(s) + Y_{23}(s) \end{bmatrix}$$

Comparing These Examples







Robot network: $$sX(s) = -LX(s)$$

Thermal network: $$sT(s) = -L(s)T(s)$$

Electrical network: $$sV(s) = -L(s)V(s)$$

Static consensus protocol:

$$x_i(s) = -\frac{1}{s}\sum_{j\in\mathcal{N}_i}[\lambda_{ij}(x_i(s) - x_j(s))]$$

Dynamic consensus protocol:

$$x_i(s) = -\frac{1}{s}\sum_{j\in\mathcal{N}_i}[\lambda_{ij}(s)(x_i(s) - x_j(s))]$$

Consensus conditions: Under some conditions on L and $L(s)$,

$$x_i(t) \to x^* \qquad T_i(t) \to T^* \qquad v_i(t) \to v^*$$

Outline

Introduction

- Systems as networks

Consensus Paradigm

- Dynamic Networks
- Concepts and extensions
- Consensus and Resilience
- Examples

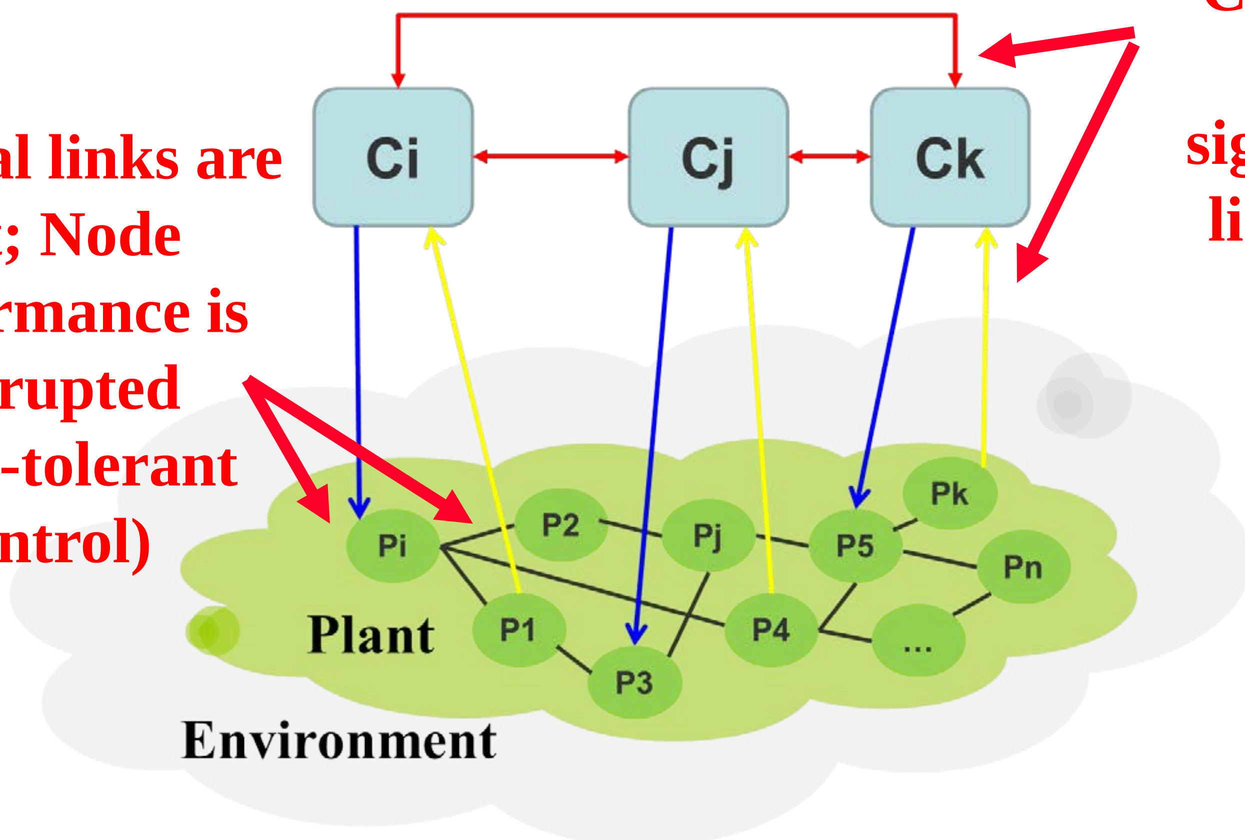
Resilient Dynamic Networks though Disturbance Attenuation

- **Designing network weights**
- **Designing network controllers**

Resilient Control as Disturbance Attenuation -1

- Consider a consensus network that models some critical system
- What could go wrong?

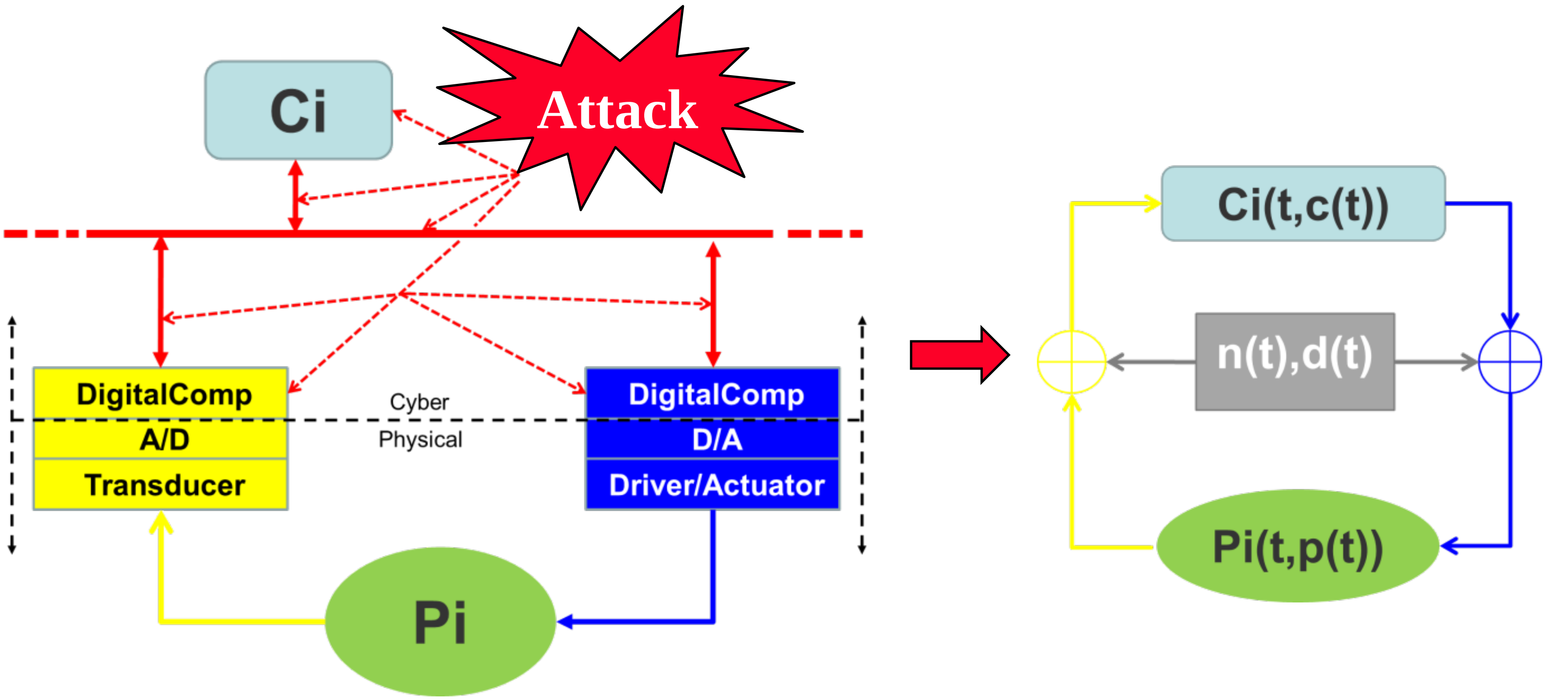
Physical links are lost; Node performance is corrupted (fault-tolerant control)



Cyber-attack corrupts signals; Cyber links are lost (robust control)

Resilient Control as Disturbance Attenuation -2

- The cyber-attack problem can be viewed as follows:



Design for Resilient Dynamic Networks

- An aspect of resilience is disturbance attenuation: keeping network as close to consensus as possible
- Will consider two disturbance attenuation problems in consensus networks:
  1. Designing network weights
  2. Designing decentralized and distributed controllers
- Will consider $\mathcal{L}_2$ disturbances with finite energy
- Also have results addressing bounded (but infinite energy) disturbances in $\mathcal{L}_\infty$

A Quick Reminder: H_∞ Norm

- For a system with an **input** $d(t)$ and an **output** $z(t)$, the H_∞ norm of the transfer function matrix from d to z ($T_{zd}(j\omega)$) is defined by:

$$\|T_{zd}(j\omega)\|_\infty = \max_{d(t)\neq 0} \frac{\|z(t)\|_2}{\|d(t)\|_2} = \max_{\omega} \sigma(T_{zd}(j\omega))$$

- We use the H_∞ norm of the $T_{zd}(j\omega)$ because it is an upper bound on the amplification of the energy in d to z:

$$\|z(t)\|_2 \leq \|T_{zd}(j\omega)\|_\infty \|d(t)\|_2 \, , \, d(t) \in \mathcal{L}_2(t)$$

1. Graph Design for Disturbance Attenuation

- Suppose our network of identical nodes is defined as

$$\begin{aligned}\dot{x}_i &= Ax_i + F\sum_{j\in\mathcal{N}_i} w_{ij}(x_j - x_i) + Ed_i, \\ z_i &= x_i - \frac{1}{N}\sum_{j=1}^{N} x_j,\end{aligned}$$

- $z_i = [z_1, z_2, \ldots, z_N]^T$ is called the "*disagreement vector*"
- $d_i = [d_1, d_2, \ldots, d_N]^T$ is the disturbance vector
- **Problem**: Pick the weights w_{ij} so that
  1. Consensus is achieved when $d_i(t) \equiv 0$
  2. $\|T_{zd}(j\omega)\|_\infty$ is as small as possible when $d_i(t) \neq 0$

Solution

Theorem

Let $\gamma > 0$ be given. The network reaches consensus when $d \equiv 0$ and $\|T_{zd}(s)\|_\infty \le \gamma$ if $\exists\ P = P^T \succ 0$ so

$$\begin{bmatrix} \Omega_2 & PE \\ E^T P & -\gamma^2 I_n \end{bmatrix} \preceq 0,$$
$$F^T P + PF \succeq 0,$$

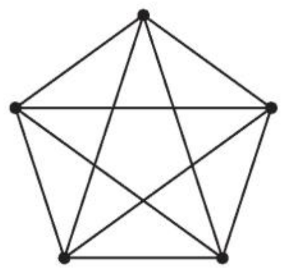
where $\Omega_2 = (A - \lambda_2 F)^T P + P(A - \lambda_2 F) + I_n$.

- λ_2 is the second smallest eigenvalue of L
- Design the weights by solving (convex optimization):

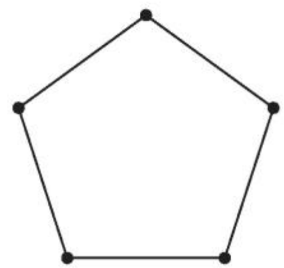
$$\begin{aligned} &\text{maximize } \lambda_2(w) \\ &\text{subject to } c^T w \le b_c \end{aligned}$$

Graph Design Example

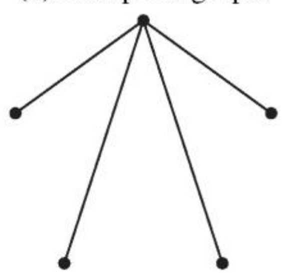
- Consider 5 different networks:



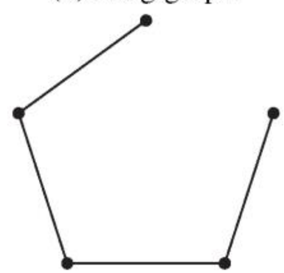
(a) Complete graph.



(b) Ring graph.



(c) Star graph.



(d) Tree graph.

- Question: what value of weights give best disturbance rejection?

Results

- Let the plant be defined by

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \; E = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \; F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

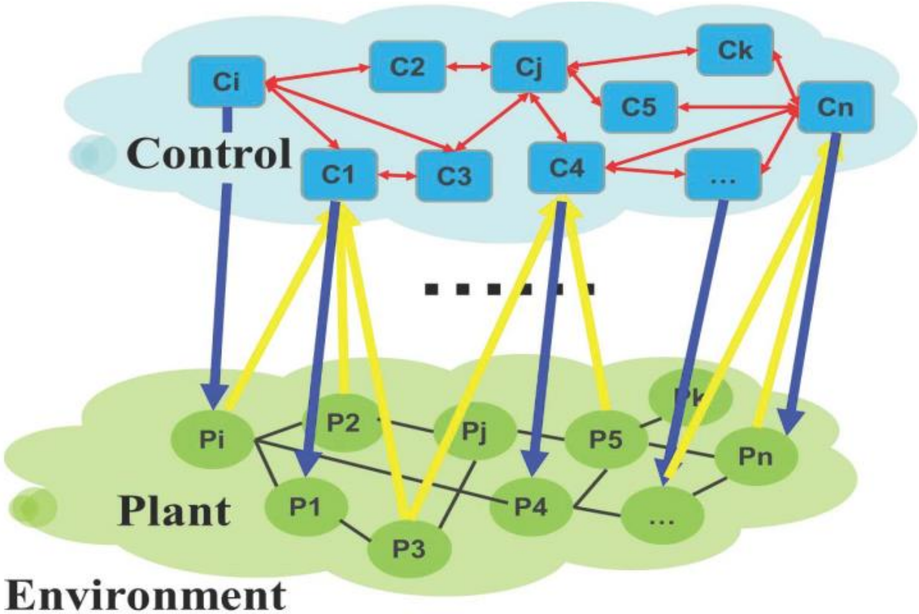
  and let the constraint be $\sum w_{ij} \leq 50$
- This produces the following

	Complete graph	Ring graph	Star graph	Tree graph
$w^\star$	$5 \times 1_{10}$	$10 \times 1_5$	$12.5 \times 1_4$	$[10\ 15\ 15\ 10]^T$
$\lambda_2^\star$	25.0	13.82	12.5	5.0
γ_{min}	0.0565	0.1021	0.1128	0.2774
$\lVert T_{zd}(s) \rVert_\infty$	0.0565	0.1021	0.1128	0.2774

- Perhaps not surprising, the complete graph is best

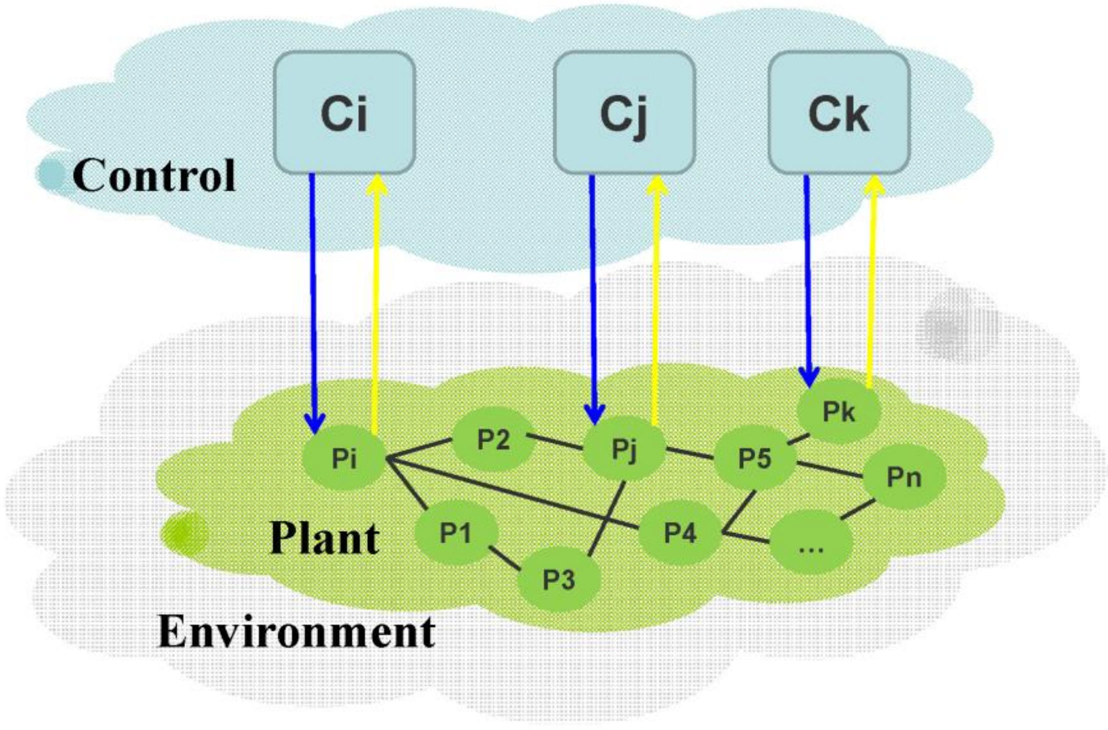
Networks Controlling Networks

- Let each node in the plant have inputs and be subject to disturbances; let the state be available as an output
- Introduce a controller network; three variants:
  - Decentralized
  - Distributed
  - Fully-interconnected (shown)



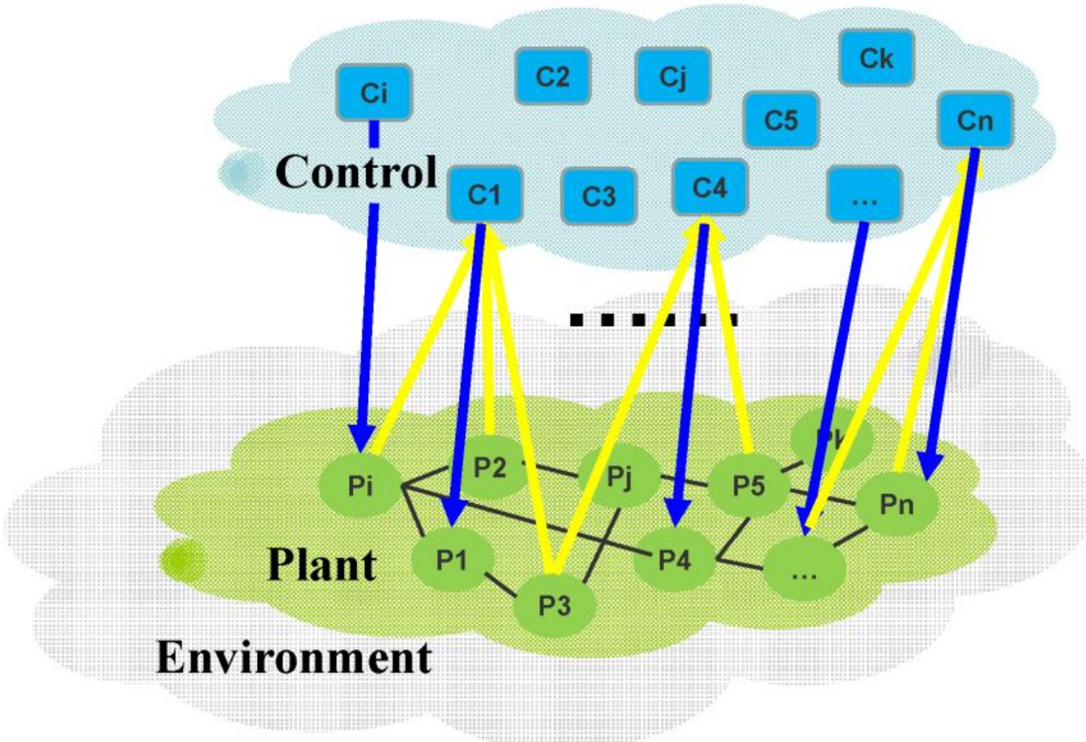
Decentralized Control

- Each node has its own controller
- Each controller uses only its own node's state



Distributed Control

- Each node has its own controller
- Each controller has a state feedback neighborhood
- Controller neighborhoods do not have to match plant neighborhoods



2. Controller Design for Disturbance Attenuation

- Consider again the same plant network, but with an added input

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i + F \sum_{j \in \mathcal{N}_i} w_{ij}(x_j - x_i) + Ed_i, \\ z_i &= x_i - \frac{1}{N} \sum_{j=1}^{N} x_j, \end{aligned}$$

- Two cases
  1. Case 1: Decentralized control: $u_i = -Kx_i$
  2. Case 2: Distributed control: $u_i = K \sum_{j \in N_{c,i}} w_{c,ij}(x_j - x_i)$
- **Problem**: Pick the gain K so that
  1. Consensus is achieved when $d_i(t) \equiv 0$
  2. $\|T_{zd}(j\omega)\|_\infty$ is as small as possible when $d_i(t) \neq 0$
- Can give similar Theorems and LMIs as above

Controller Design Example

- Consider same plant as above, with inputs added to each node
- Plant network is a ring graph with unity weights
- Results become
  1. Decentralized controller: $\|T_{zd}\|_{\infty} = 0.0389$
  2. Distributed controller with different controller topologies:

	Complete graph	Ring graph	Star graph	Tree graph
w_c	1_{10}	1_5	1_4	1_4
$\lambda_{c,2}$	5	1.382	1	0.382
K	$\begin{bmatrix} 0.1908 & 34.5966 \\ 34.7841 & -0.1740 \end{bmatrix}$	$\begin{bmatrix} 0.6425 & 34.4697 \\ 35.0695 & -0.5626 \end{bmatrix}$	$\begin{bmatrix} 0.9244 & 34.4434 \\ 35.3193 & -0.8597 \end{bmatrix}$	$\begin{bmatrix} 2.7760 & 33.8264 \\ 36.2309 & -2.5066 \end{bmatrix}$
γ_{min}	0.0439	0.0835	0.0982	0.1592
$\|T_{zd}(s)\|_{\infty}$	0.0095	0.0286	0.0440	0.0862

- **Comment**: Decentralized controller does better than some distributed topologies

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- Designing network weights
- Designing network controllers

What's next? -1

- Apply ideas from control theory in a network context
  - Controllability/observability/fault-tolerance
  - Note: New journal:

IEEE Transactions on Control of Network Systems

… systems with interconnected components ...

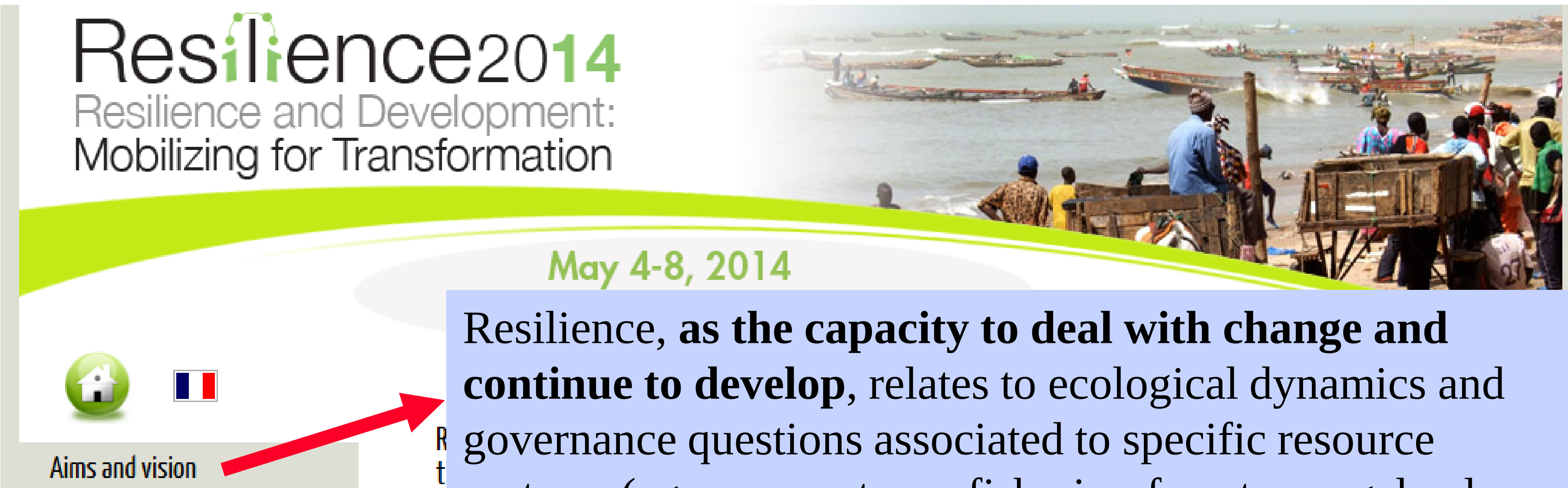
The IEEE Transactions on Control of Network Systems publishes high-quality papers on systems with interconnected components. The journal is primarily interested in problems related to the control

What's next? -2

- Apply ideas from graph theory and network science
  - Degree distributions, clustering, centrality, betweenness, communicability, …
  - E.g., betweenness centrality: the number of shortest paths from all vertices to all others that pass through that node
    - These two graphs will have different vulnerabilities

What’s next? -3

- Apply ideas from ecology (http://www.resilience2014.org)



Resilience, **as the capacity to deal with change and continue to develop**, relates to ecological dynamics and governance questions associated to specific resource systems (agro-ecosystems, fisheries, forests, rangelands, marine and freshwater ecosystems), and to global issues such as biodiversity conservation, urban growth, economic development, human security and well- being.

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Thanks for your attention!

Questions?