The UNIVERSITY of OKLAHOMA
Mewbourne School of Petroleum and Geological Engineering

Hydraulic Fracturing in a Heterogeneous Reservoir

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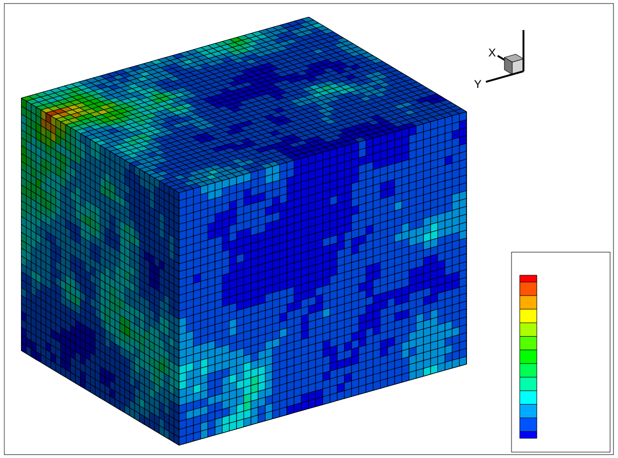
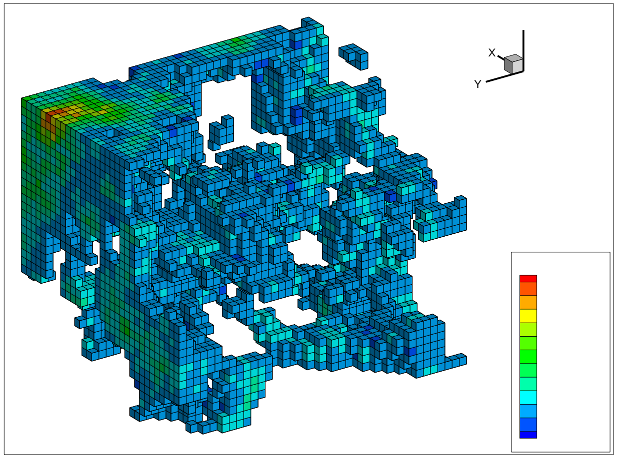
Outline

- Introduction
- Motivation
- Objectives
- Methodology
- Results and Discussion
- Conclusions

Motivation

- Heterogeneities occur in geological formations, they are caused by rock itself, or by the existence of discontinuities.
- In this study, we treat the heterogeneous reservoir as a continuous media but with different mechanical properties, and then evaluate their effects on reservoir activities. This work focuses on hydraulic fracturing.





Gauss Random Distribution of Young’s Modulus

Objectives

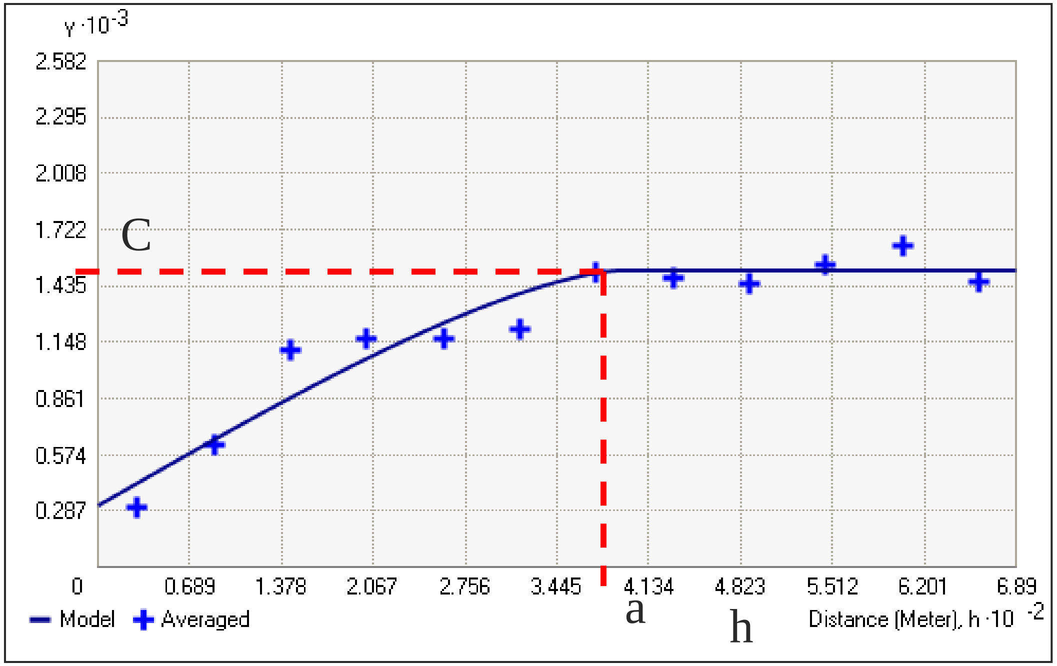
- Consider heterogeneity using a geostatistical model
- Simulate hydraulic fracture in heterogeneous reservoirs
- Analysis the effect of heterogeneity on hydraulic fracturing and reservoir behaviors during stimulation

Generation of Heterogeneous Fields

Exponential Semivariogram Model :

$$\gamma(h) = C\left[1 - exp\left(\frac{-(3h)^2}{a^2}\right)\right]$$

where:
h: distance between two points
a: Correlation lengths.



Transform normal distribution parameters (mean value and standard deviation) into corresponding log normal parameters:

$$\sigma^2_{\ln E} = ln\left(1 + \left(\frac{\sigma_E}{\mu_E}\right)^2\right)$$

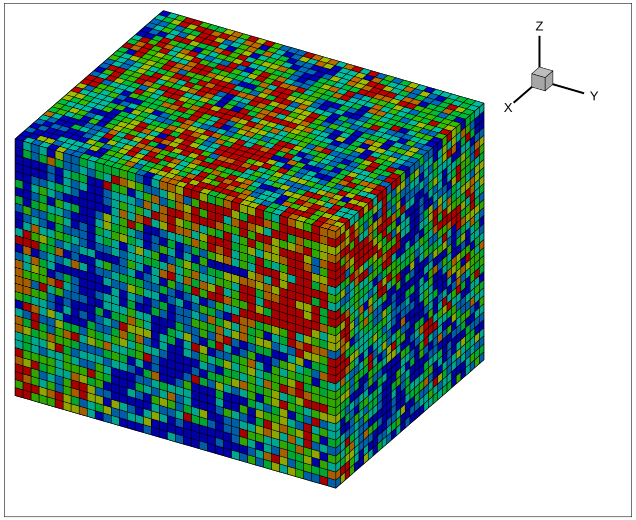
$$\mu_{\ln E} = \ln(\mu_E) - \frac{1}{2}\sigma^2_{\ln E}$$

From log normal distribution to actual normal distribution:

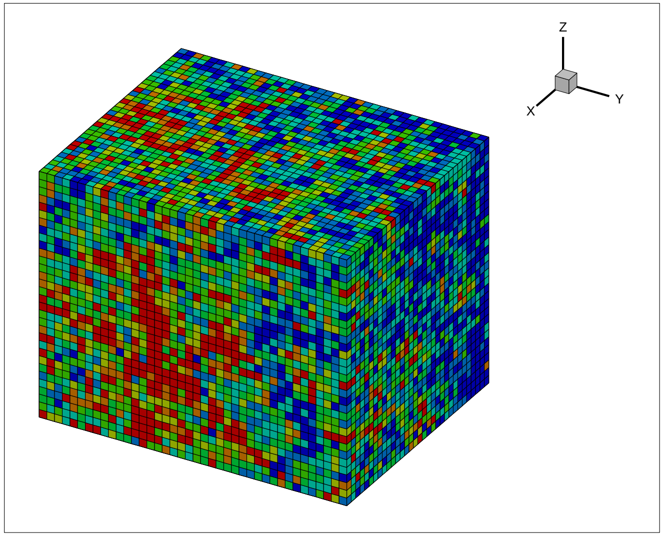
$$E_i = exp(\mu_{Ln\,E} + \sigma_{\ln E} G_i)$$

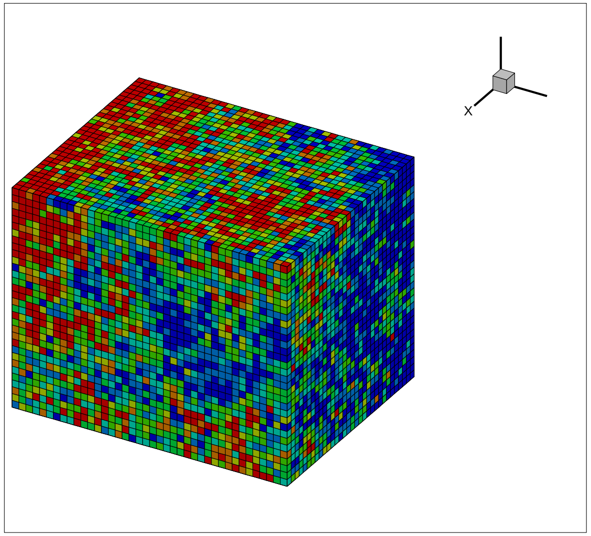
Generation of Heterogeneous Fields

- Conditional Gauss Random Distribution with a same correlation length



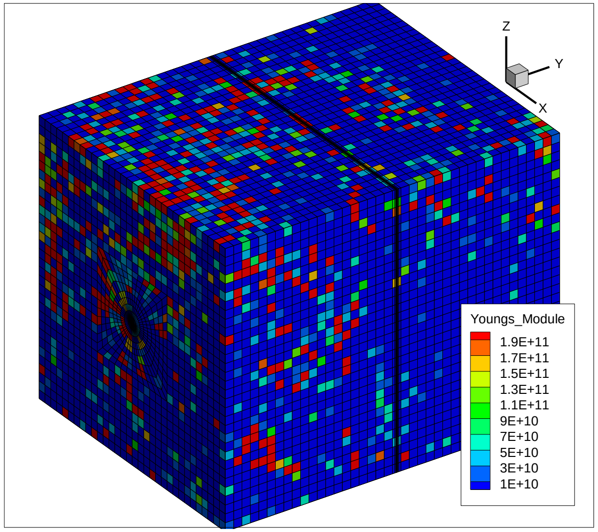
Grid size is 20 m, the correlation length is 40 m.



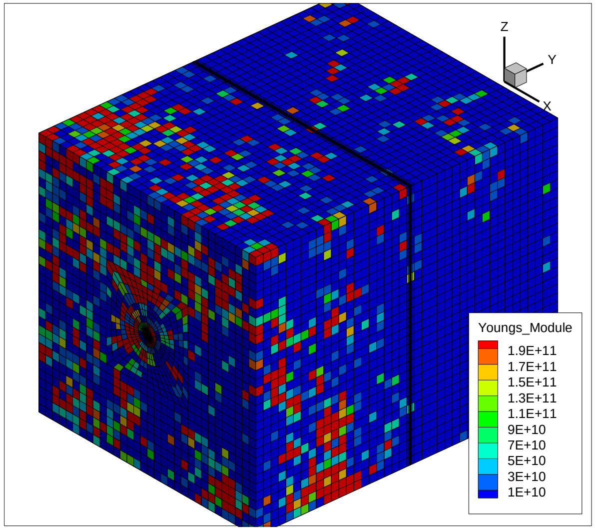


Generation of Heterogeneous Fields

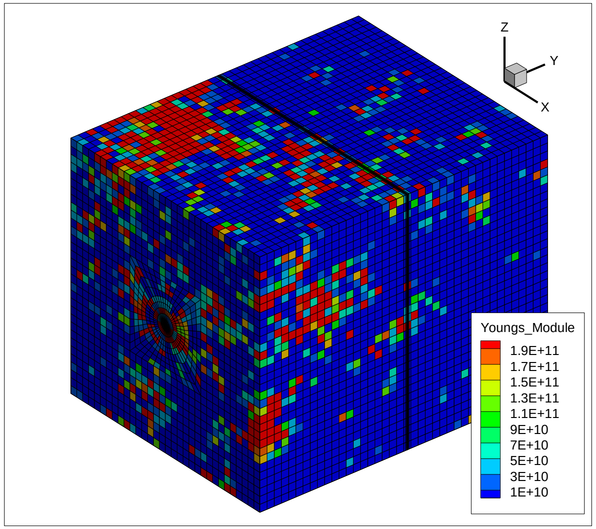
- Conditional Gauss Random Distribution with different correlation lengths



Correlation length: 20 m



Correlation length: 40 m



Correlation length: 80 m

Poroelastic Model

Field Equations

$$G\nabla^2 u_i + \frac{G}{1-2\nu} u_{k,ki} - \alpha p_{,i} = -F_i$$

$$\frac{\partial p}{\partial t} - \kappa M \nabla^2 p = -\alpha M \frac{\partial \varepsilon_{kk}}{\partial t} + M\left(\gamma - \kappa f_{i,i}\right).$$

Boundary Conditions

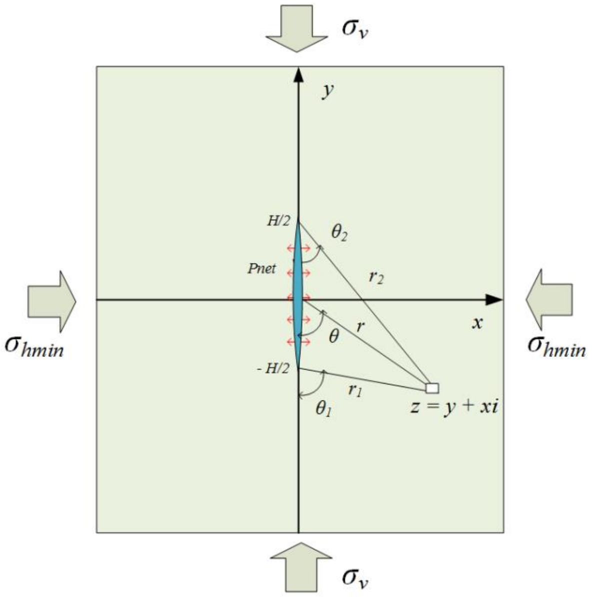
Mode 1

$$\sigma_n(x,y,z,t) = -H(t),$$
$$p(x,y,z,t) = 0;$$

Mode 2

$$\sigma_n(x,y,z,t) = 0,$$
$$p(x,y,z,t) = H(t).$$

H(t) denotes the Heaviside step functic



Mode 1+2

$$F = \left(p_f - \sigma_0\right)F_1 + \left(p_f - p_0\right)F_2$$

(Carter and Booker,1980)

Finite Element Discretization

Spatial integration

$$[\mathbf{k}_m]\{\mathbf{u}\} + [\mathbf{c}]\{\mathbf{p}_w\} = \{\mathbf{f}\}$$

$$[\mathbf{c}]^{\mathrm{T}}\left\{\frac{d\mathbf{u}}{dt}\right\} - [\mathbf{k}_c]\{\mathbf{p}_w\} - [\mathbf{S}]\left\{\frac{\partial \mathbf{p}_w}{\partial t}\right\} = Q$$

Temporal discretization

$$\begin{bmatrix} \mathbf{k}_m & \mathbf{c} \\ \mathbf{c}^{\mathrm{T}} & -(\mathbf{S} + \theta\Delta t\mathbf{k}_c) \end{bmatrix}\begin{Bmatrix} \Delta u \\ \Delta p \end{Bmatrix} = \begin{Bmatrix} \Delta f \\ \Delta Q + \Delta t\mathbf{k}_c\mathbf{p}_{t_{n-1}} \end{Bmatrix}$$

(Linear interpolation in time using the Crank-Nicolson approximation)

Pressurized HF in Poroelastic Rock

Stress State

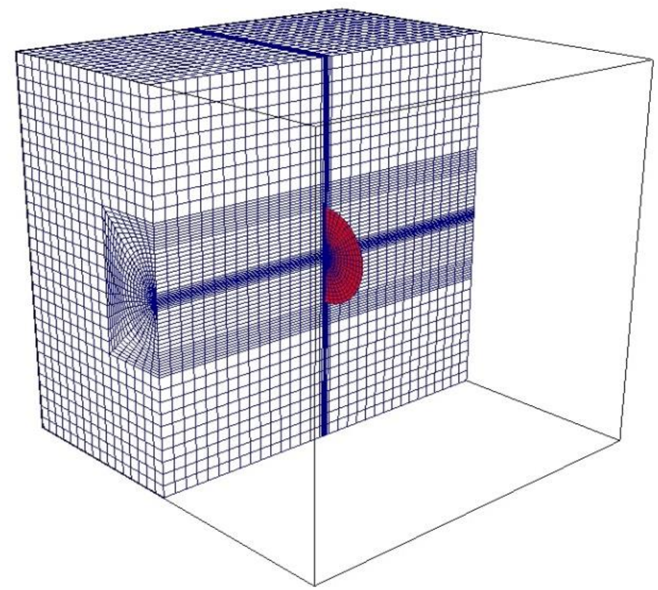
Vertical Stress:	50 MPa	Initial Pore Press.:	18 MPa
Max. Hori. Stress:	33 MPa	Net Press.:	7 MPa
Min. Hori. Stress:	29 MPa		

Rock Properties for Homogeneous Case

Drained Poisson's Ratio:	0.15
Undrained Poisson's Ratio:	0.29
Biot coefficient:	0.7
Young's Modulus:	2.76×10^{10}Pa
Permeability:	5.0 md
Fluid Viscosity:	2.0×10^{-4}Pa

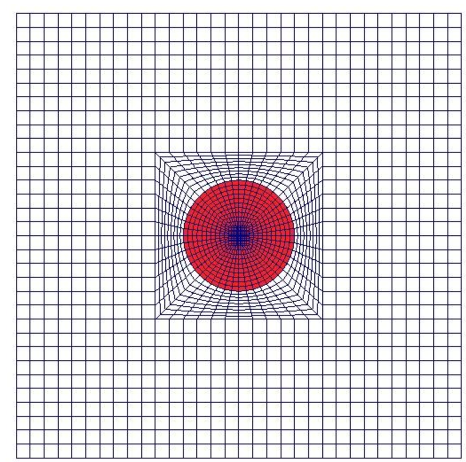
Boundary Conditions:

Four Lateral Boundaries:	No displacement Constant pore press.
Top Boundary:	No constraints on disp. No flow boudnary.



Height □ Length □ Width: 640 □ 800 □ 640 m

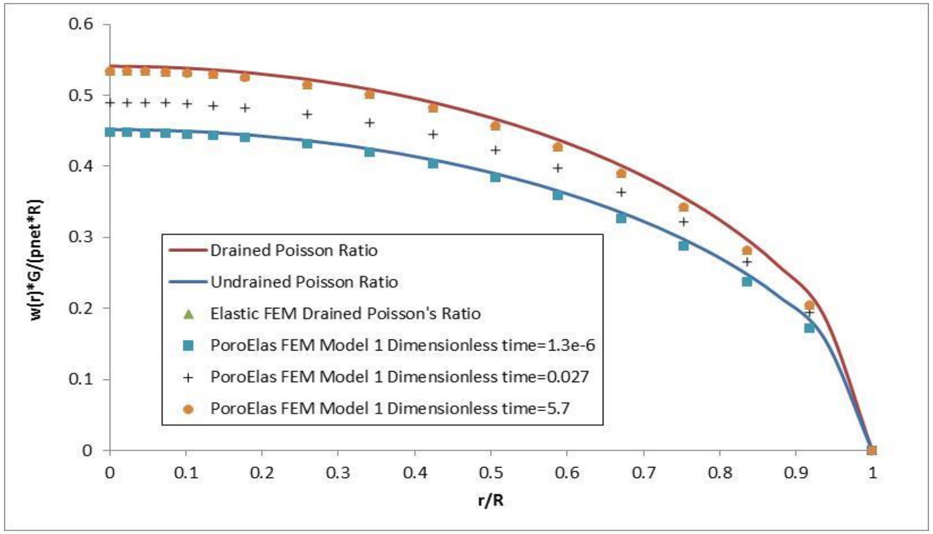
Fracture radius: 80 m



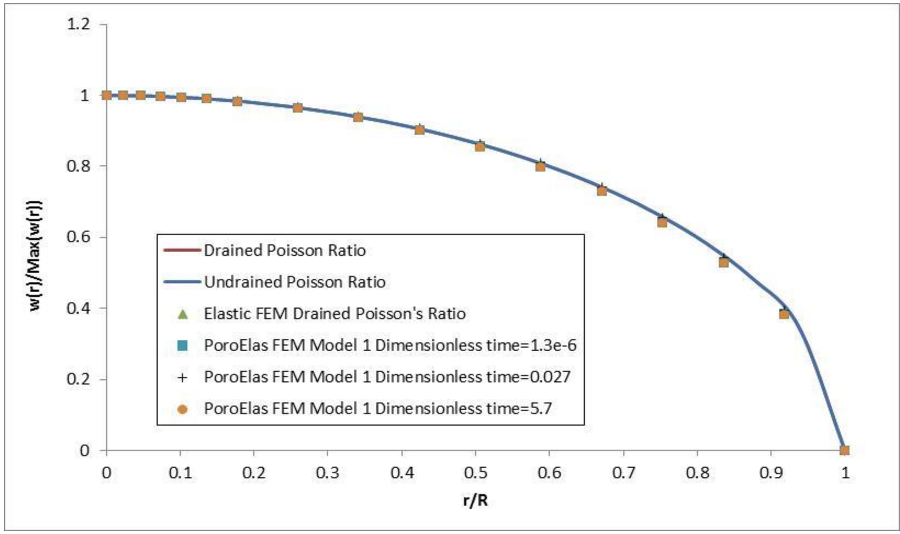
Verification of Poroealstic Model

Fracture width of uniformly pressurized fracture:

$$w(r) = \frac{2p_{net}(1-\nu)R}{\pi G}\sqrt{1-\left(\frac{r}{R}\right)^2} \qquad \text{(Sneddon 1946)}$$

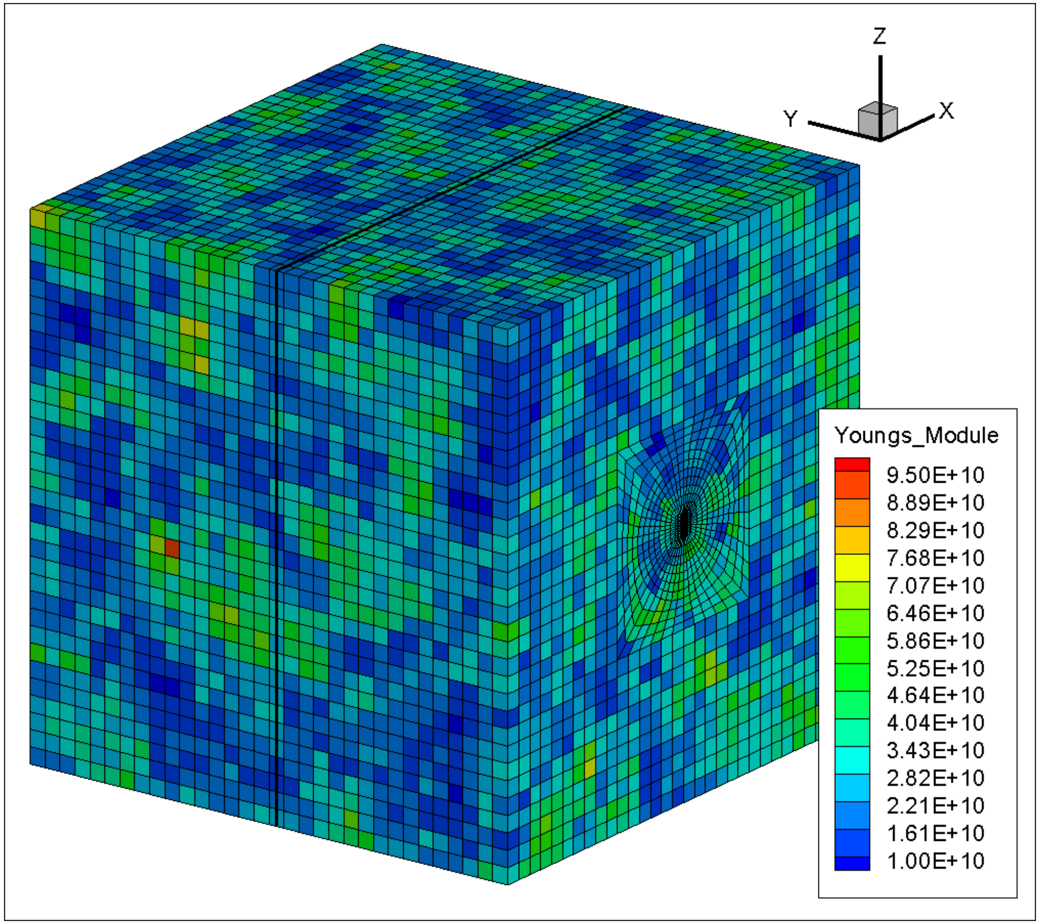


Model 1 fracture width vs. radial distance for a penny-shaped fracture.



Normalized Model 1 fracture width vs. radial distance for a penny-shaped fracture.

Young's Modulus Distribution



Input Data

Mean value: 0.276 E+11Pa

Standard Deviation: 0.138 E+11Pa

Output Statistic Data

Simulation No.: 1

Seed No.: 1

arithmetic average: 0.26E+11 Pa

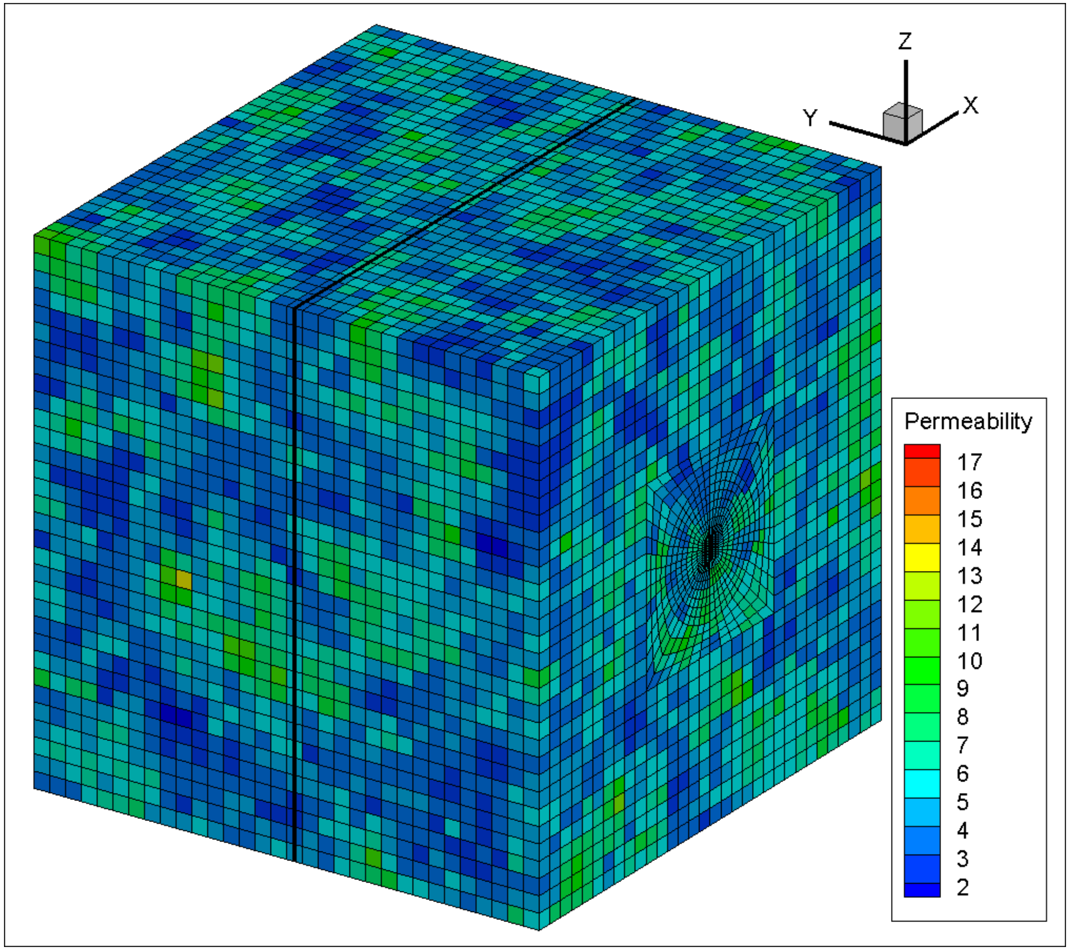
geometric average: 0.25E+11

geometric average: 0.23E+11

Max. Value: 0.10E+12

Min. Value: 0.53E+10

Permeability Distribution



Input Data

Mean value: 5.0 md

Standard Deviation: 2.0 md

Output Statistic Data

Simulation No.: 1

Seed No.: 1

arithmetic average: 0.489E+01

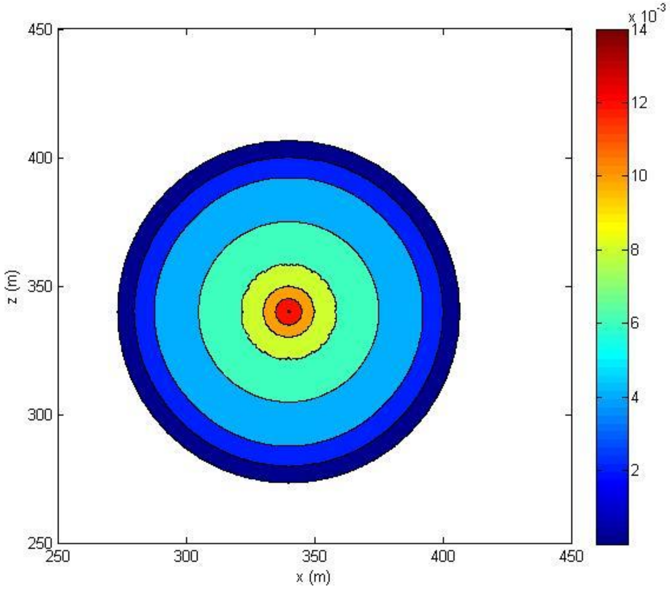
geometric average: 0.46E+01

geometric average: 0.44E+01

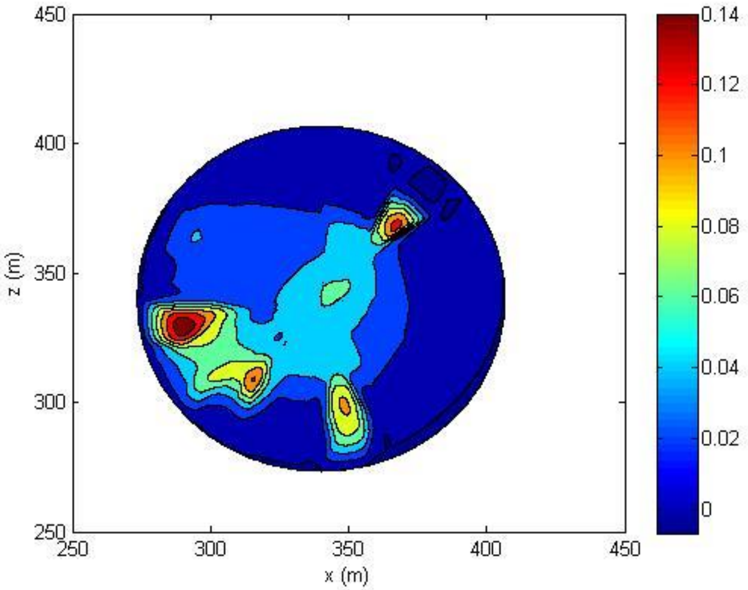
Max. Value: 0.15E+02

Min. Value: 0.13E+01

Displacement Distribution After 3 Days of Pressurization



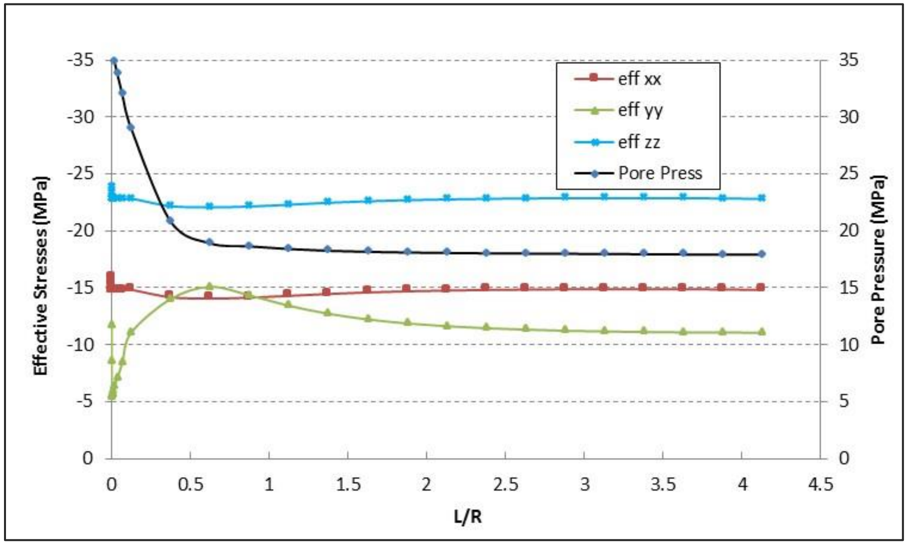
Homogeneous Case

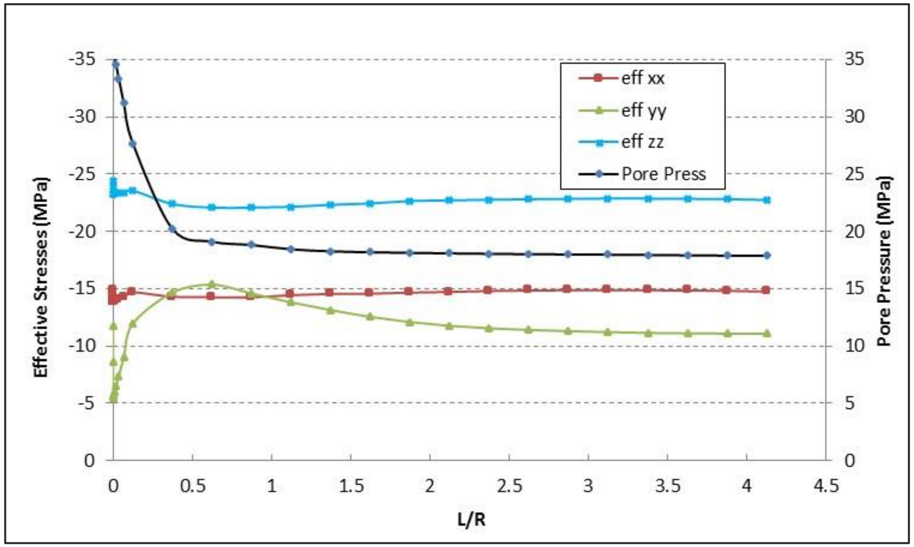


Heterogeneous Case

Displacement in minimum horizontal stress direction on pressurized fracture surface

Stress and Pore Pressure Evolution Around Pressurized Fracture



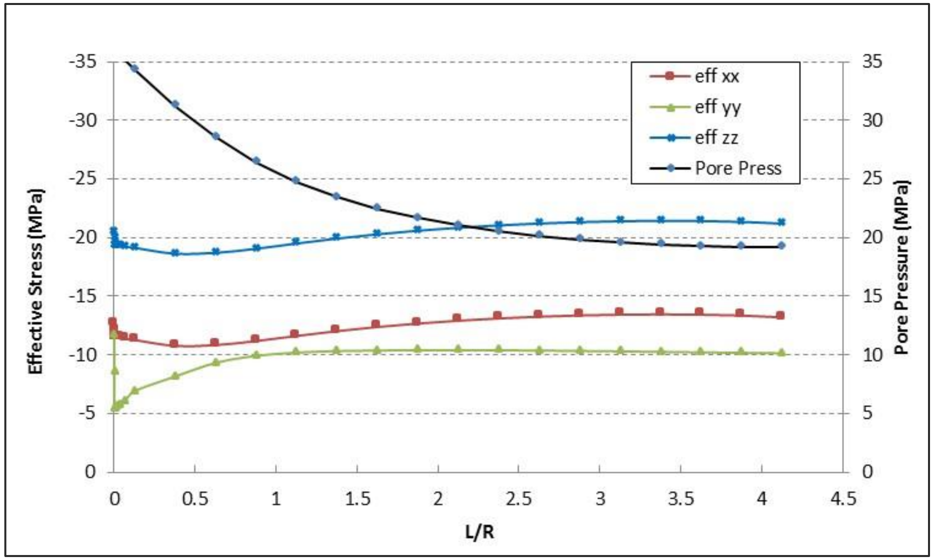


Homogeneous Case

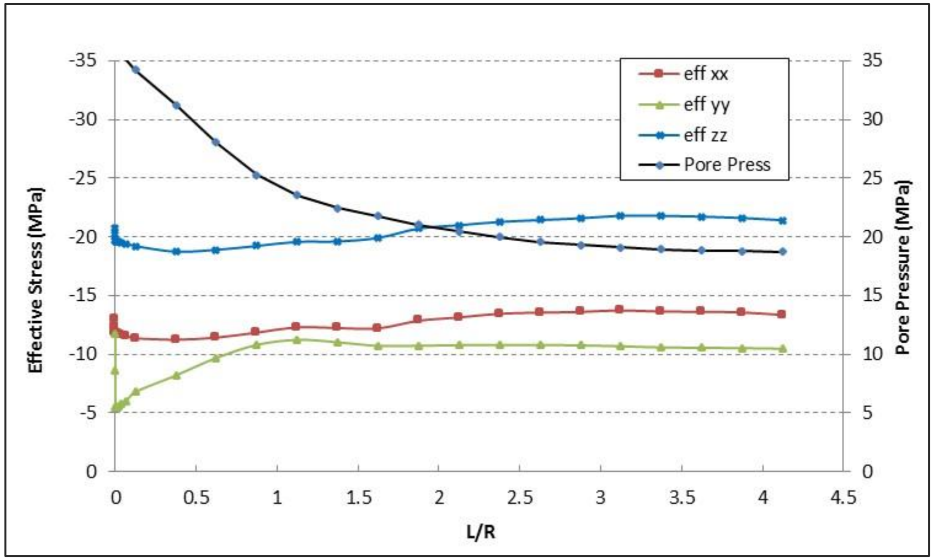
Heterogeneous Case

S_{xx} S_{yy} S_{zz} distribution at time = 7 mins

Stress and Pore Pressure Evolution around Pressurized Fracture



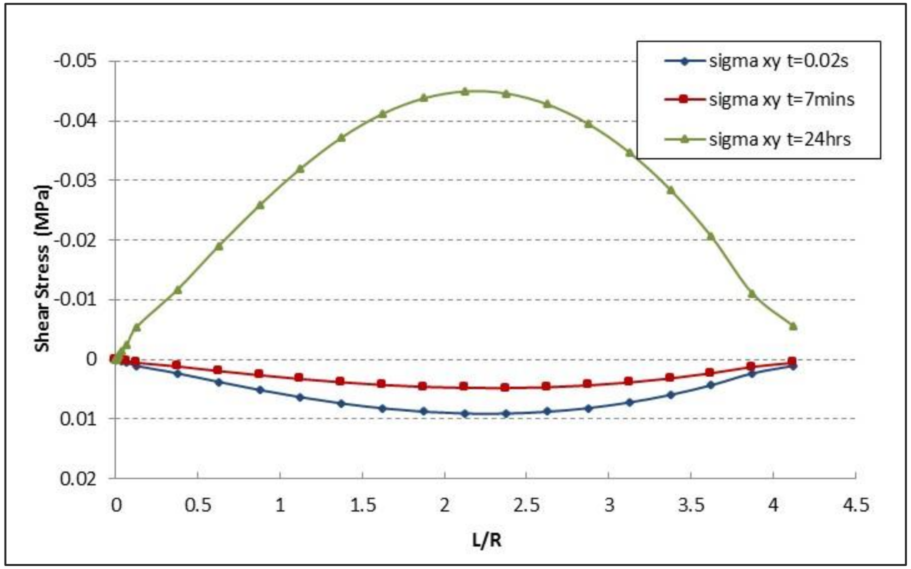
Homogeneous Case

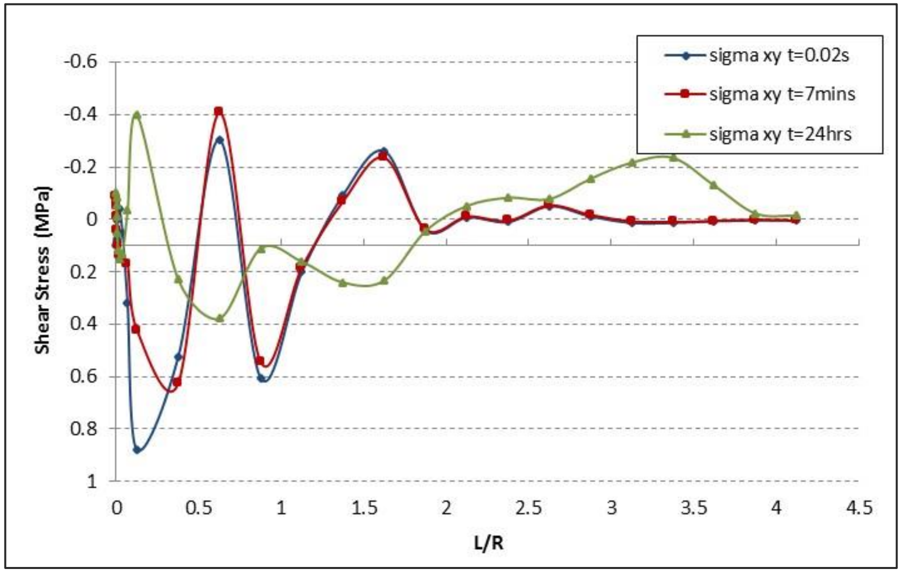


Heterogeneous Case

S_{xx} S_{yy} S_{zz} distribution at time = 24 hours

Stress Evolution around Pressurized Fracture



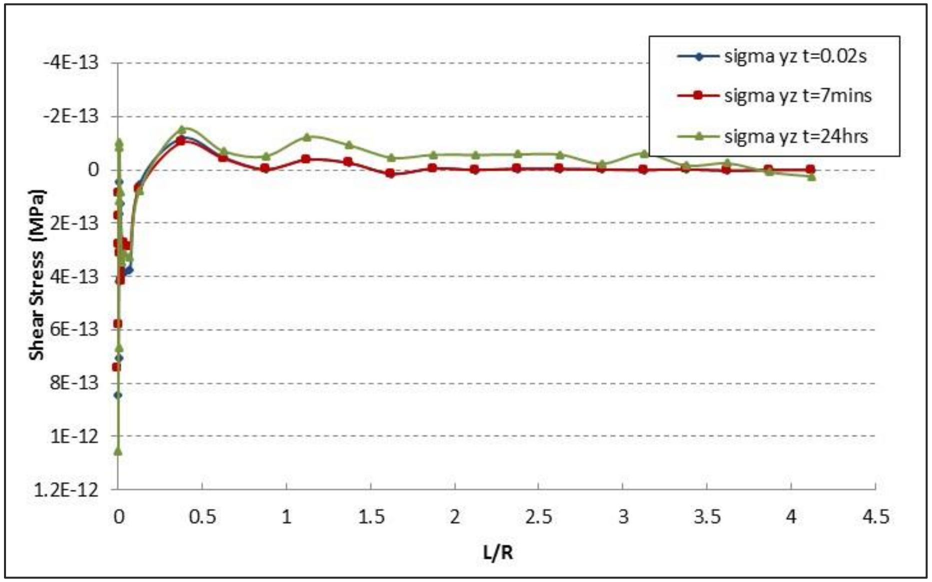


Homogeneous Case

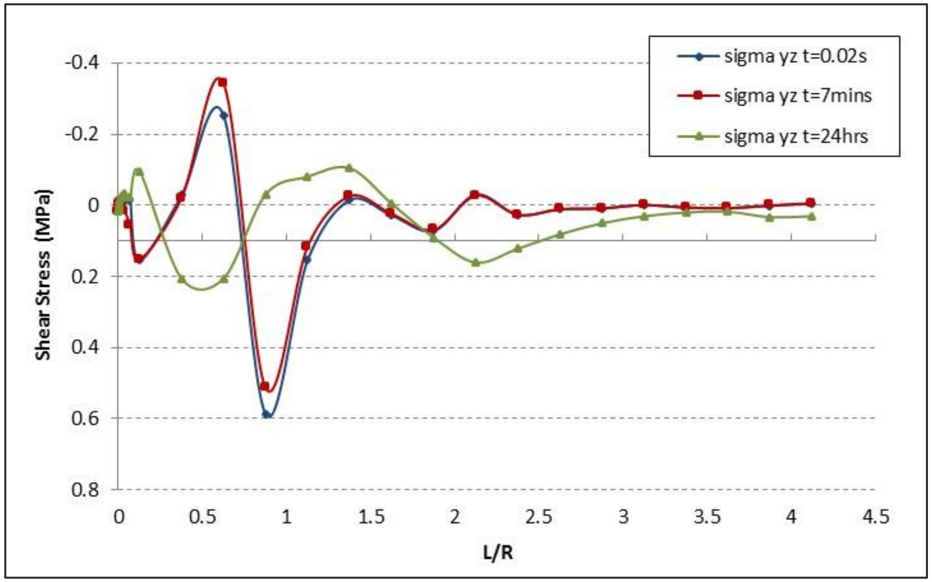
Heterogeneous Case

Shear Stress S_{xy}

Stress Evolution around Pressurized Fracture



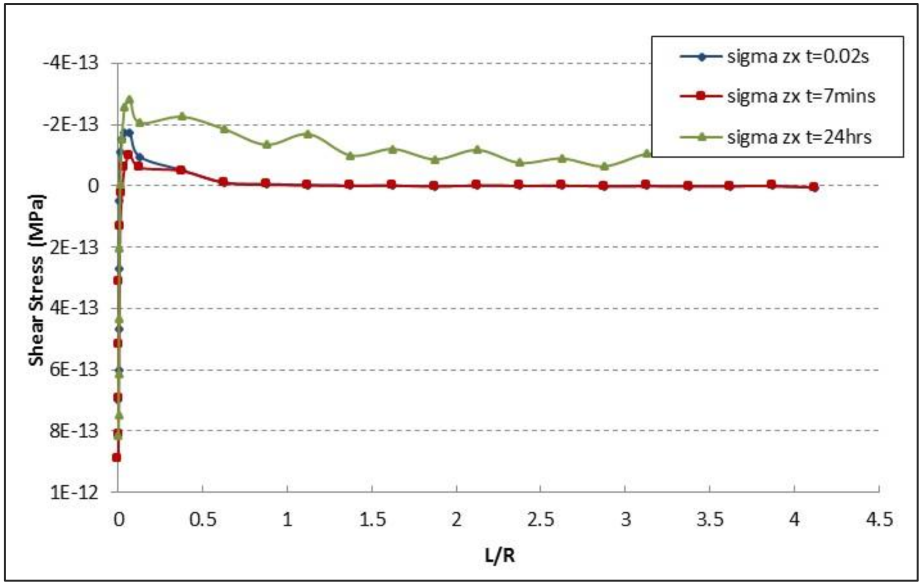
Homogeneous Case

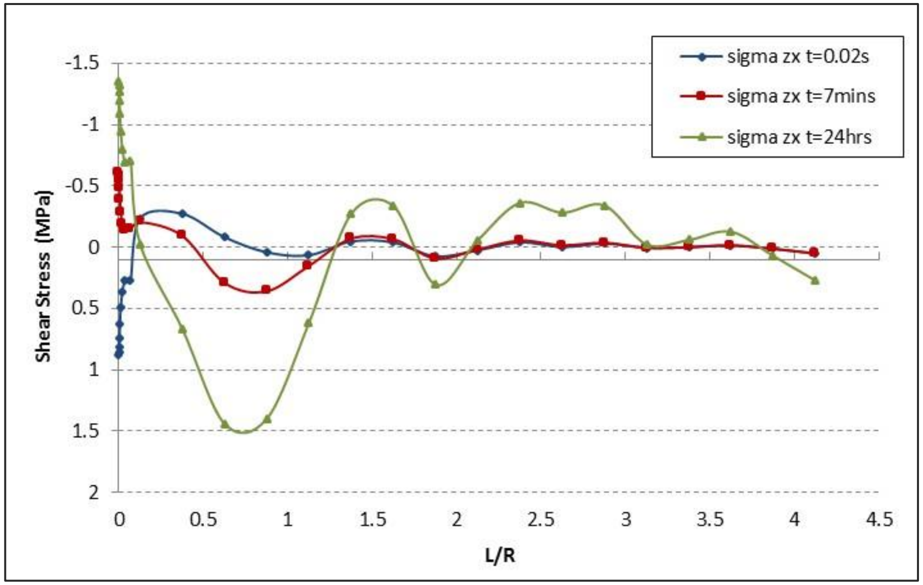


Heterogeneous Case

Shear Stress S_{yz}

Stress Evolution Around Pressurized Fracture



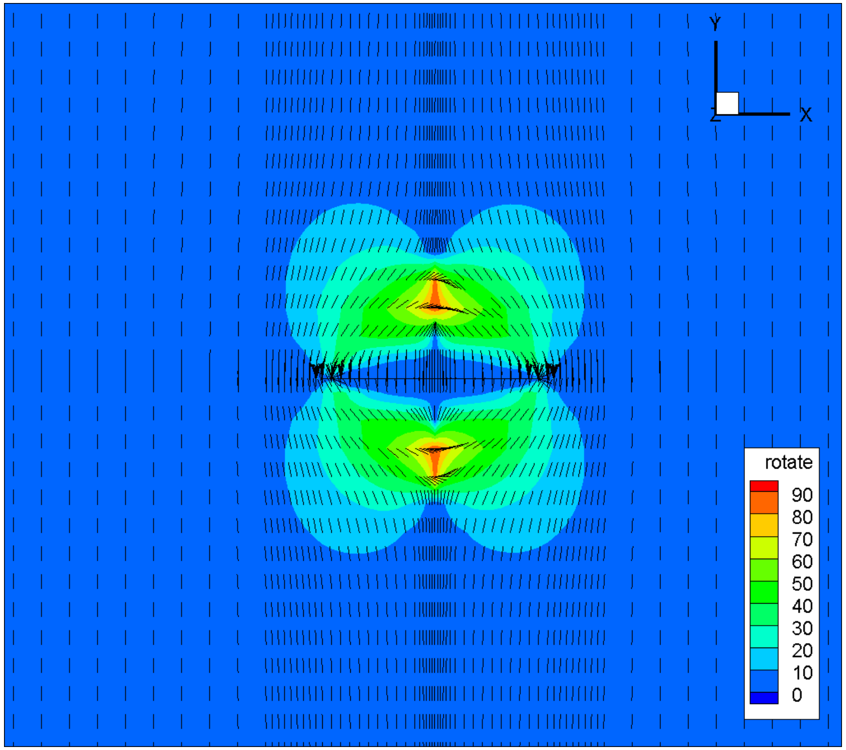


Homogeneous Case

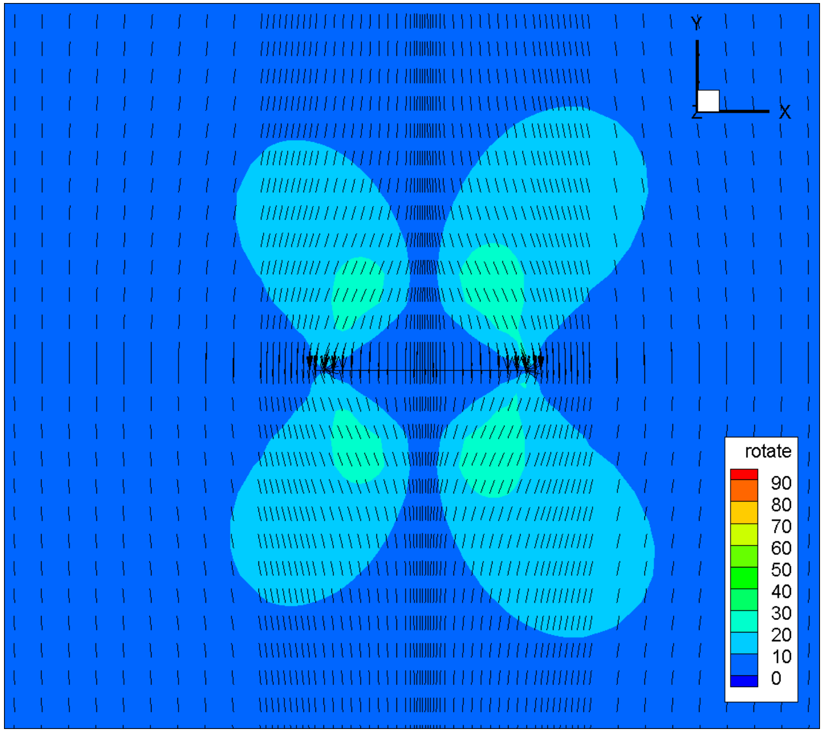
Heterogeneous Case

Shear Stress S_{zx}

Reorientation of Minimum Principal Stress



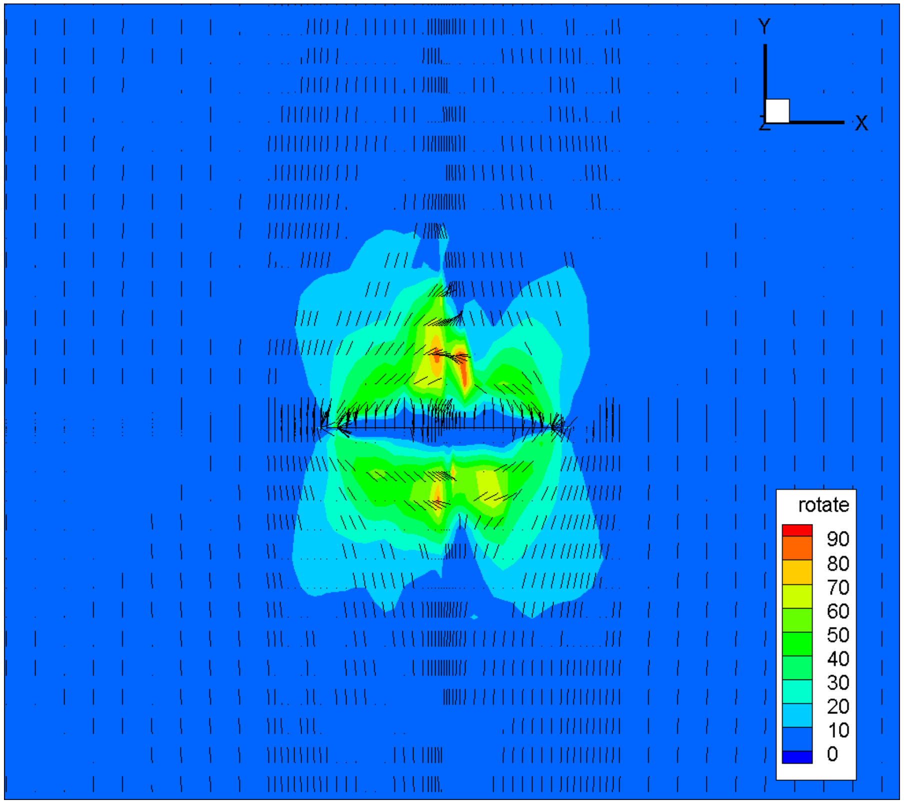
Time = 7 mins



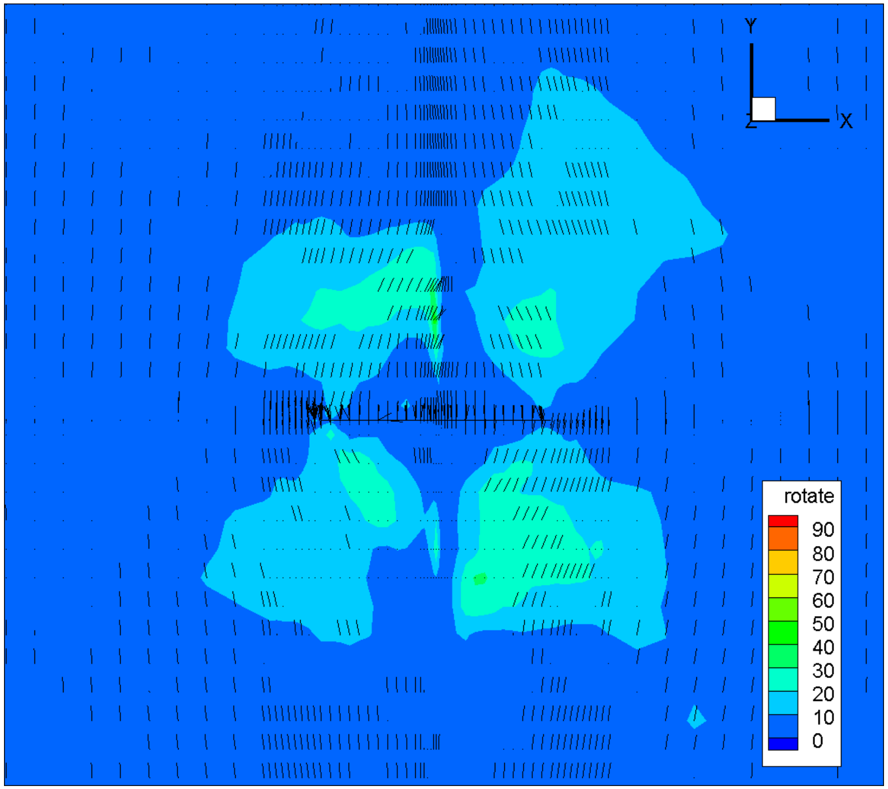
Time = 24 hrs

Homogeneous Case

Reorientation of Minimum Principal Stress



Time = 7 mins



Time = 24 hrs

Heterogeneous Case

Coulomb Stress Distribution

- Coulomb Criterion

$$\boxed{\tau = S_0 + \mu\sigma_n}$$

- Shear and Normal Stresses

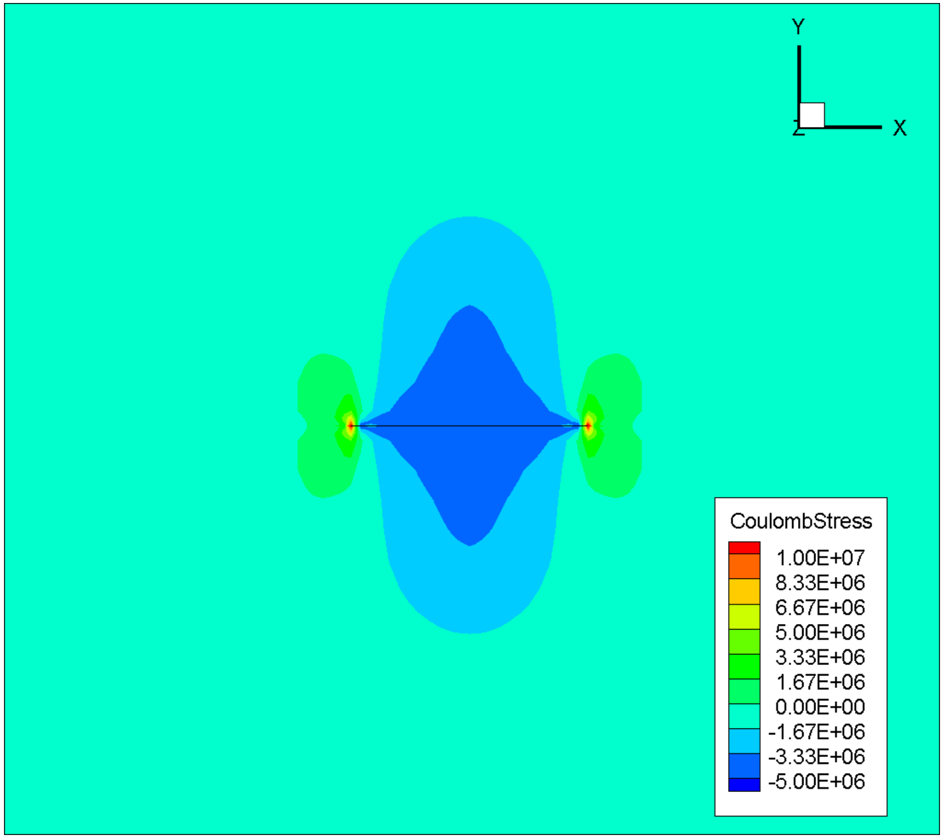
$$\sigma_{n(\beta)} = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)cos2\beta$$

$$\tau_{(\beta)} = \frac{1}{2}(\sigma_1 - \sigma_3)sin2\beta$$

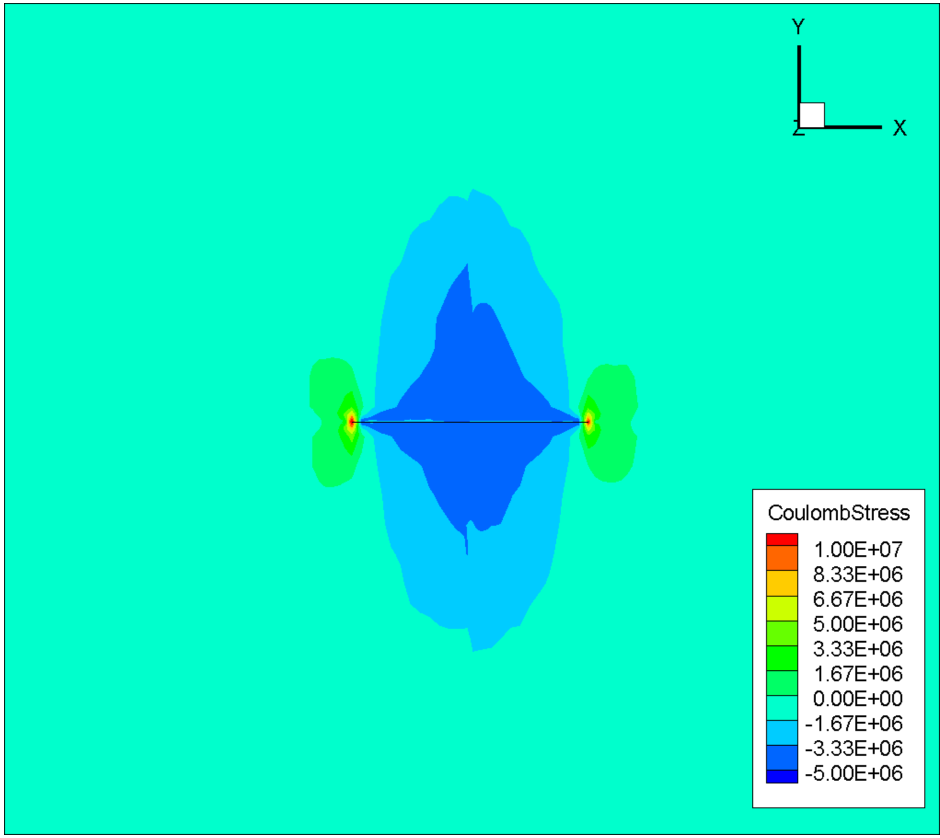
- Coulomb Stress

$$\boxed{MCStress = \frac{1}{2}(\sigma_1 - \sigma_3)(1 + \mu^2)^{\frac{1}{2}} - \frac{1}{2}(\sigma_1 + \sigma_3)\mu - P_0 - S_0}$$

Coulomb Stress Distribution



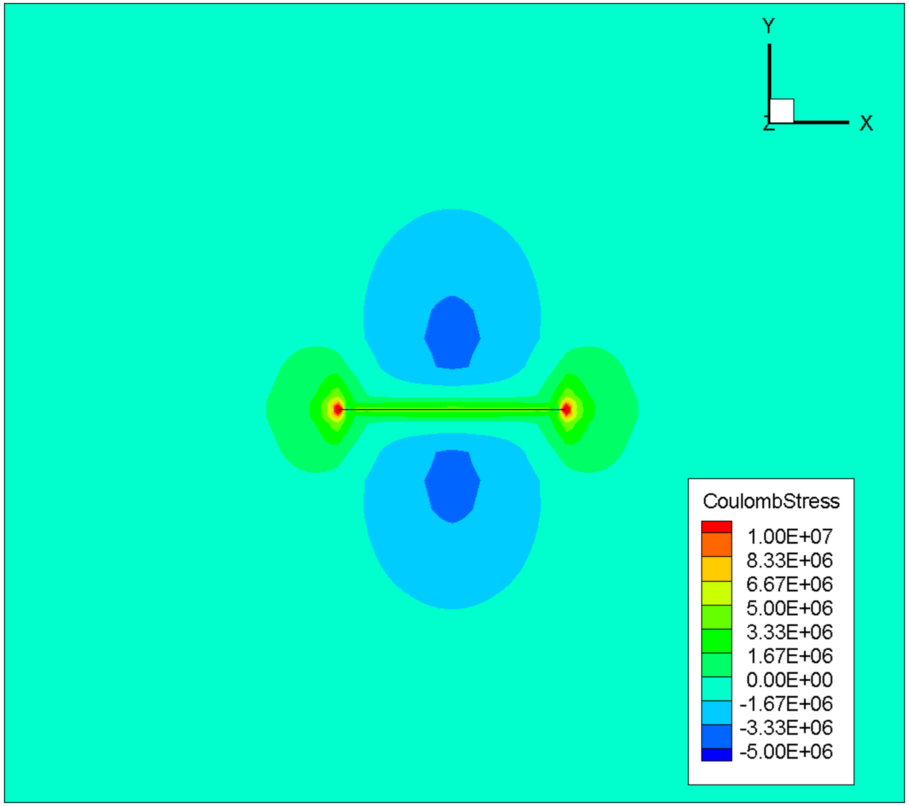
Homogeneous Case



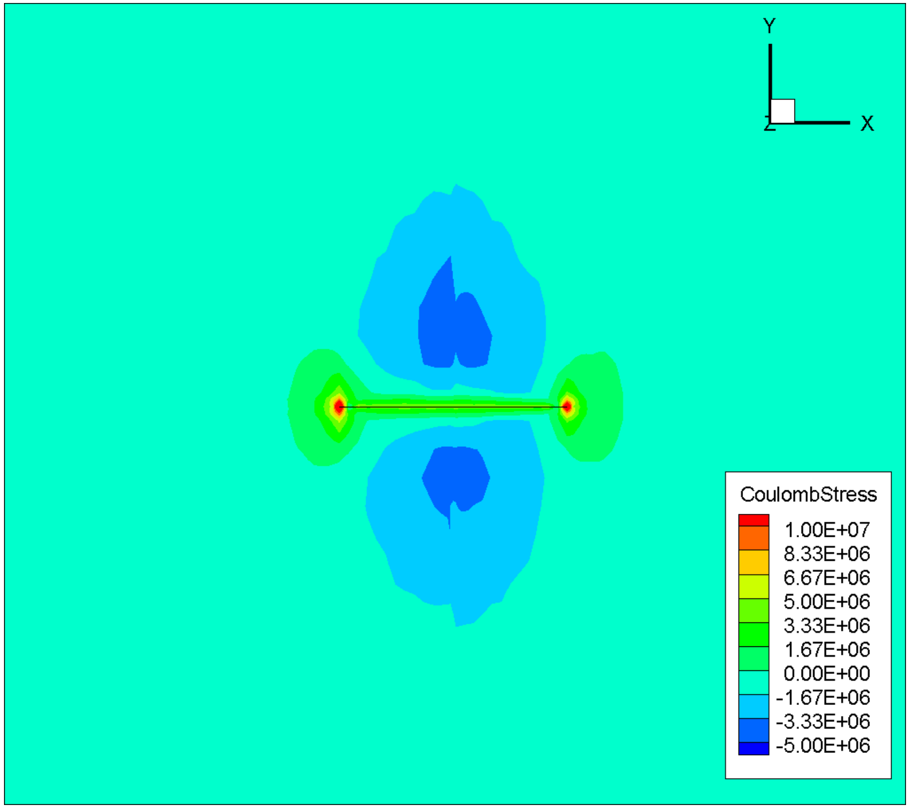
Heterogeneous Case

Time = 0.02 s

Coulomb Stress Distribution



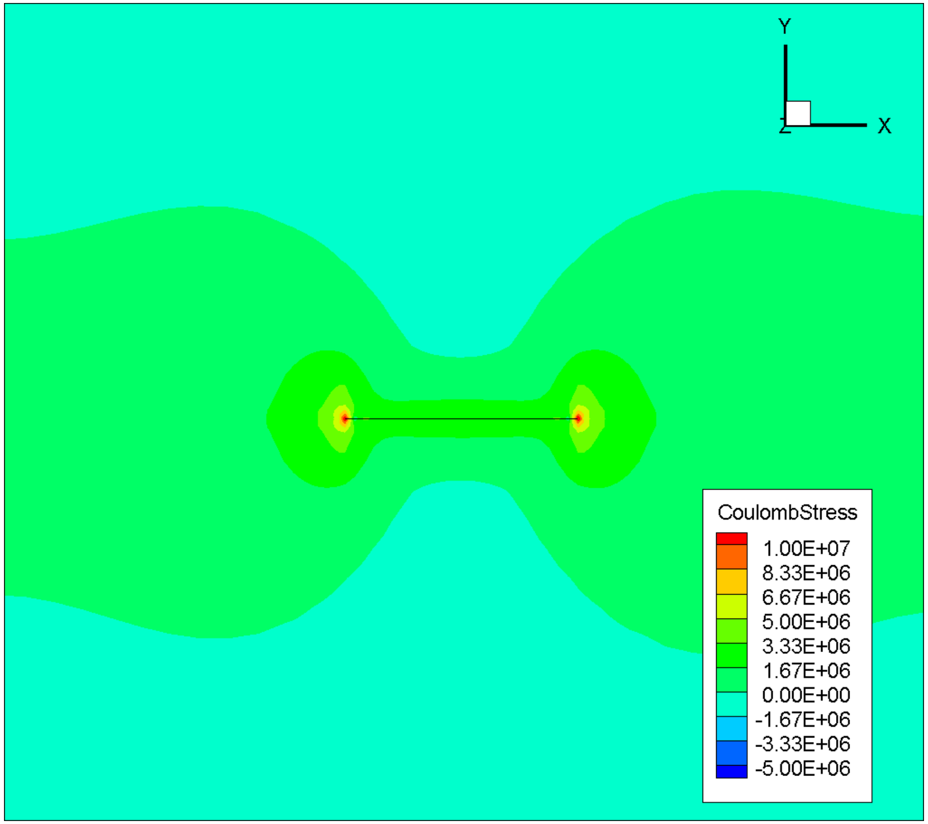
Homogeneous Case



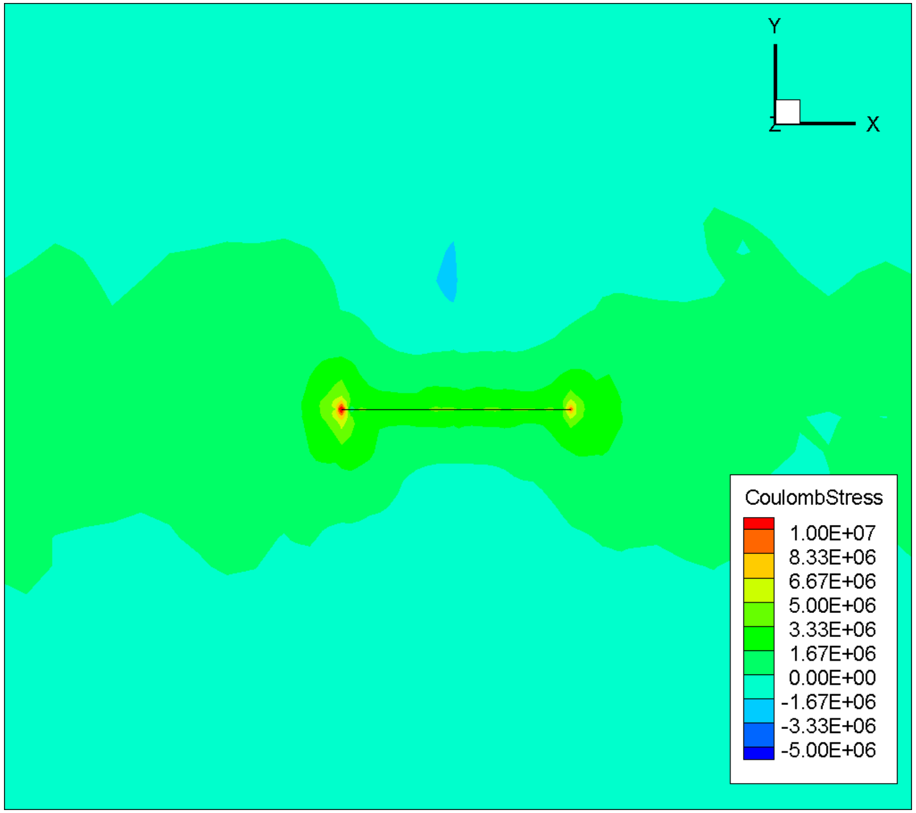
Heterogeneous Case

Time = 7 mins

Coulomb Stress Distribution



Homogeneous Case



Heterogeneous Case

Time = 24 hours

Conclusions

- The induced normal stresses (ΔSxx, ΔSyy, ΔSzz) and pore pressure have similar magnitudes for homogeneous and heterogeneous porous media;
- However, the induced shear stresses (ΔSxy, ΔSyz, ΔSzx) in heterogeneous rock are approximately one order of magnitude larger than those in homogeneous rock;
- The induced stresses in heterogeneous case make the directions of principal stress rotate randomly, which may cause hydraulic fractures to propagate in complex manner;
- In homogeneous case, the horizontal stresses will also rotate due to the pressurization of hydraulic fracture, but they are still in the horizontal plane.

Conclusions

- In the beginning of pressurization, the regions beside the fracture surface are stable. The failure region extends along fracture tips;
- Due to pore pressure diffusion into formation, the regions beside the fracture surface gradually change from stable to unstable status. The size of stress shadow will gradually decrease, and may not exist in porous media for long-term pressurization.

Thank you!